ADAPTED ALGORITHMIC PRESCRIPTIONS FOR THE CARRYING OUT OF SPECIFIC MATHEMATICAL ACTIVITIES

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ABSTRACT

The article discusses an adapted algorithm of a generalized model of systematized activities. The concrete application is defined by the specifics of the activities performed when solving problems of construction. An illustrative example has been given.

ASSISTING THE GUESSING

Let us begin with a modification of the problem that was Edison’s favorite one.

Write down twelve different possibilities for using one object (for instance - a pencil).

The more and different possibilities for the use of a pencil you can write down, the more resourceful you are. Try it yourself.
How many different applications did you write?
With collective efforts, in even less time, we can write down more applications.
For example:

- for writing, for drawing (in any way- for leaving a mark on paper);
- for stirring a liquid;
- for dialing a phone number;
- for making a hole in paper,
- for defense;
- for filling up holes in furniture;
- for stopping the movement of a door or a window;
- as a conductor;
- for fuel;
- as a means serving didactic purposes when teaching “Cylinder”, “Prism”, “Cone”, “A Directed Segment”;
- for advertising something;

• for drawing a direct line, i.e. as a drawing tool;
• as a pointer;
• for knocking;
• as a hook for hanging things on it, etc.

What is the collective success due to?

Different persons direct their thoughts in different directions when thinking of the possibilities for the use of the pencil on the basis of its characteristics. Consequently, this search could be directed. The pencil is a solid object; it has a definite form and size, a special core, a specially processed and formed end. Each of the specified applications of the pencil is based on one or another of its characteristics. But any of these applications can be made possible by using another object that has the same characteristics.

Consequently, in order that we become more quick-witted, it is sufficient to look for different possibilities for the use of the pencil based on its characteristics taken in different configurations, while looking at these as at equivalent ones.

Let us discuss one further problem.

Prove that if it is true for a triangle that $m_c = \frac{1}{2}c$, *then the triangle is rectangular.*

In order to find different ways for solving this problem, one looks for different ways to prove that an angle is right. The field, in which the search is executed, is a restricted one within the set of sufficient conditions making it possible to recognize that an angle is a right one. These sufficient conditions have been systematized in the didactic system of indications (DSI) for the notion *right angle* [1]. This analytical logical centre is actually the point of support for the heuristic activity relating to the search and the finding out of different solutions for the same problem. When solving this problem, the heuristic role of the didactic system of indications is used to execute the activity of recognizing (bringing under a notion) the right angle.

The didactic systems of indications play a heuristic part also with regard to the problems of construction because different sufficient conditions of the respective didactic system of indications are used in the argumentation of the constructions.

Similarly to the preceding problems, we can place the problem of finding different structures when solving a problem of construction, because with such type of problems we do not recognize but rather construct objects making use of the scope (volume) of some notion.

The formation of the heuristic thinking with the students is a very essential component in the educational and cultivating activity. The student has to obtain a sum of knowledge, to acquire skills for deductive thinking and considerations, but also the mathematical intuition of the student has to be developed, the habits of an independent search for regularities and objective laws have to be created and developed. The students have to be introduced to sufficient general methods for a purposeful search of solutions to problems (proofs of theorems or structures for the solutions of problems of construction), i.e. *to create and form methods and ways that do not depend on what the type of the problem is or which section of the educational contents the problem refers to*.

What is common in the two problems is the search for a point of support for the algorithmic presentation of the activities, relating to the finding of different solutions for the same problem, i.e. *in the directing and management of the guessing*. This algorithmic presentation and management of the specified activities is possible due to the purposeful search, restricted within definite limits that have been set by way of systematizing the knowledge on the notion. This is the common method and way of procedure that we shall observe when looking for different solutions of the basic problems of construction.

MOTIVES

Knowledge is acquired in order to be applied in the practice. The knowledge in mathematics is used in solving problems. For the purpose of finding out different solutions to one problem, knowledge from different sections of mathematics is used. Therefore, it is only natural to accept that guessing depends on the subjective experience of the person solving problems in the field of mathematics. This is the reason why the preliminary theoretical knowledge and the solving of many problems is a necessary prerequisite for the guessing.

Besides, it has been said in [2] that the guessing depends on the organization and the way in which the knowledge is stored and kept in the long-term memory. On the other hand, the guessing also depends on the systematization of the different ways in which a certain activity can be executed [5]. Following [2] and [5], it becomes obvious that *the best organization is the systematization of knowledge and activities so that all the components of this knowledge and all the elements of the activities acquire the same potential of usability, thus avoiding some of them to dominate for the account of others*.

Following [3], there exist five ways to teach a person how to solve problems. The student:

- ✓ Is being taught algorithms for solving problems;
- ✓ *Is being taught algorithms for searching algorithms;*
- ✓ Is being taught general methods for searching solutions, which methods are of non-algorithmic nature;
- ✓ Is being taught independent rules for acting by pointing to him/her which of these can be applied in the process of solving a problem;
- ✓ Is not being taught algorithms nor methods of non-algorithmic nature, nor any rules but rather, when confronted with the problems, he/she is practically placed into a problematic situation and then he/she relies on the independent finding out of the algorithmic procedures.

ESSENCE

On the grounds of the description of these methods and ways [3], we express the opinion that for the formation of skills for solving problems, the second one is the most suitable because the very ways of solving problems represent algorithms.

The essence of this approach consists in the fact that the persons taught are provided an algorithm for searching algorithms for the solving of problems. In this case, attempts are needed but these attempts are purposeful, they have a definite objective, and the actions of the person solving the problem are *fully determined by the algorithmic prescriptions*.

A universal method for the activities of the student is given in [5]. The actions of the student x when carrying out any mathematical activity y, for the purpose of which he or she has knowledge on the ways $p_1, p_2, \ldots, p_k$, is modeled in the following way:

$$p_1(x,y) \vee p_2(x,y) \vee \ldots \vee p_k(x,y) \to p(x,y).$$

In the following part of the paper we shall discuss an adaptation of this universal model made on the basis of the specific language and the specific activity of the students relating to this type of problems.

A systematization has been made here called *"generalized algorithm-prescriptions for the constructing of ..."*, because the basic objects are actually the problems of construction. The term *"algorithm-prescriptions"* has been taken from L. Landa [3]. The motives are as follows.

It has been specified in [3] that processes and events can be described by means of an algorithm if these are profoundly well known and vice versa. The availability of this algorithmic instruction also presupposes the existence of a possibility for algorithmic instruction on how to search for solutions in case of problems of proving but also in case of problems of constructions and also in case of problems of calculation. These general approaches provide support for guessing when searching for solutions to a problem notwithstanding of what type it is. *When the methods of action are known, then they acquire objectification in different forms of wording adapted to the respective type of problems.*

The motives specified here above in [1] and [2] justify the use of the term "generalized". The generalization provides for the transfer of knowledge from one field to another. Analogical algorithmic instructions can be applied with regard to the theoretical knowledge but also with regard to the activities adequate to the theoretical knowledge.

The categorical form accepted for the presentation of the algorithm-instruction suggests that this means provides for the conditions that might bring about a good result in the case, when the necessary preliminary theoretical knowledge is available. The potential success of the search is possible because in that way a purposeful search has been ensured. Therefore, the general heuristic scheme used for finding different solutions for the discussed basic problems of construction can be called **method of localization**.

The separate components of the algorithmic instruction are disjunctively connected because they are independent. This logical structure provides the possibility for variation in the search and finding out of solutions and these attempts, as we already pointed out, have been fully predetermined.

APPLICATION

The methods specified have been applied for the finding out of solutions for problems of construction. An attempt has been made to systematize the solutions found and to show how different solutions can be found for the basic problems of structuring on the basis of the generalized information on the basic notion in the problem.

Let us discuss the following concrete example:

The generalized algorithm-prescription for construction of geometrical objects specified here below appears as an adaptation (according to the type of solving a problem) of the logical model, as described in [5].

A generalized algorithm-instruction for constructing of the midpoint of a segment

The midpoint of a segment is constructed as follows:

p_1) the segment is divided into two segments with equal lengths

or

p_2) by constructing the point of intersection of the segment and its perpendicular bisector**,**

or

p_3) by constructing the point of intersection of the diagonals of a parallelogram, in which the said segment is actually one of the diagonals,

or

p_4) in the image or inverse image of the midpoint of a segment in the case of congruence, homothetia or in case of parallel projection, whereby the said segment is the inverse image or the image of the segment, the midpoint of which is known,

or

p_5) as a point of intersection of the straight line defined by the point of intersection of the diagonals of a trapezium and the point of intersection of the continuation of the arms with the bases of the trapezium (the said segment is one of the bases of the trapezium); as a point of intersection of the straight line defined or by the point of intersection of the diagonals or by the point of intersection of the continuations of the arms and the midpoint of one of the bases of the trapezium with the other base of the trapezium (the given segment),

or

p_6) as a point of intersection of a chord (the given segment) with the diameter of a circle, perpendicular to the chord.

BASIC PROBLEM

To construct the midpoint of a given segment $[AB]$.

First way

a symbolic construction ***a graphic construction***

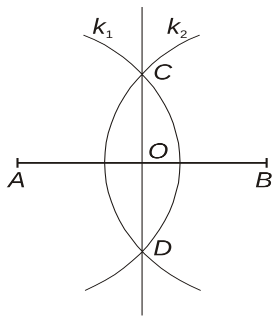
1) $k_1(A,\ r),\ r > \frac{AB}{2}$

2) $k_2(B,\ r)$

3) $k_1 \cap k_2 = \{C,\ D\}$

4) $(CD) \cap [AB] = \{O\}$

Point *O* is the point we are looking for, because it is the point of intersection of the given segment with its perpendicular bisector (p_2).



Drawing 1

It follows from condition p_3) that in order to construct the midpoint of a segment, it is sufficient to construct the point of intersection of the diagonals of a parallelogram, one of the diagonals of which is the given segment. This gives us the idea of looking for different ways to construct the parallelogram with a diagonal being the given segment.

The structure of the parallelogram can be executed with a ruler and a wheel -pan but also with the use of a bilateral ruler. If we use a bilateral ruler, then we have to apply twice the axiom specific for it, so that the ends of the given segment appear as two opposite vertices of a parallelogram.

Different solutions of this structure have been given in the following two solutions of the basic problem discussed.

Second way

a symbolic construction ***a graphic construction***

1) $m \parallel n,\ A \in m,\ B \in n$

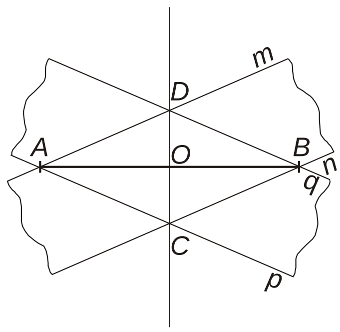
2) $p \parallel q,\ A \in p,\ B \in q$

3) $n \cap p = \{C\}$

4) $m \cap q = \{D\}$

5) (CD)

6) $(CD) \cap [AB] = \{O\}$



Drawing 2

The structure is made by a bilateral ruler. This is possible, when the length of the segment is bigger than the width of the ruler.

The theoretical basis of this construction is based on the theorem that the diagonals of the parallelogram are mutually dividing each other into two parts. The given segment represents the diagonal of the parallelogram *ACBD* and according to p_3) it follows that the point *O* is the midpoint of the segment $[AB]$.

Third way

a symbolic construction

1) $C \notin (AB)$, $CA = b$, $CB = a$

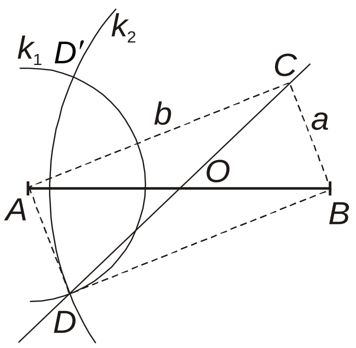
2) $k_1(A,\ a)$

3) $k_2(B,\ b)$

4) $k_1 \cap k_2 = \{D,\ D'\}$

5) $(CD) \cap [AB] = \{O\}$

a graphic construction



Drawing 3

The quadrilateral *ADBC* is a parallelogram because its opposite sides are equal. Consequently, the point *O* is the midpoint of the given segment as per p_3).

Note. Between the two points *D* and D', we choose the one which is in different half-planes with the point *C* with regard to (AB).

It follows from condition p_5) of the generalized algorithm-instruction for constructing the midpoint of the segment that in order to construct the midpoint of a given segment, it is sufficient to construct a trapezium, in which the given segment is one of its bases. For this trapezium, we have had available and known the point of intersection of the diagonals and the point of intersection of the continuations of the arms or one of these two points and the midpoint of the base, which is not the given segment. The constructing can be done by means of a ruler and a wheel-pan, but also by means of a bilateral ruler by observing the axioms of these tools of drawing. Different executions of this structure have been given in the following solution of the basic problem discussed.

Fourth way

a symbolic construction

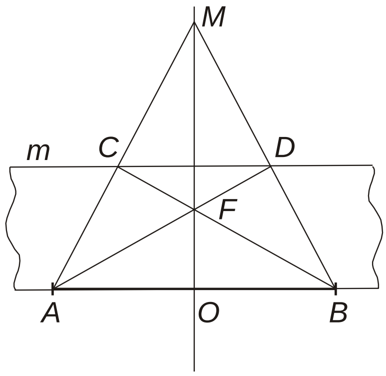
1) $m \parallel (AB)$

2) $C \in m, D \in m, (AC) \cap (BD) = \{M\}$

3) $(CB) \cap (AD) = \{F\}$

4) $(MF) \cap [AB] = \{O\}$

a graphic construction



Drawing 5

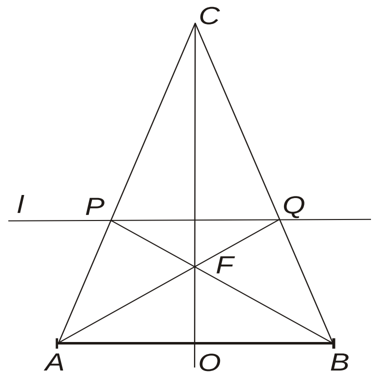
The parallel straight lines have been constructed by using a bilateral ruler in this solution. This structure is based on the theorem of Steiner, wherefrom it follows that *O* is the point looked for – the midpoint of the segment.

Note: If (AC) and (BD) cross each other on that half-plane, which does not contain *m*, or if they are parallel, then we can choose another couple of points *C* and *D*, so that the two straight lines cross and it is on the half-plane, which contains *m* or else construct by means of the bilateral ruler a new straight line parallel to (AB) in the other half-plane in relation to (AB).

Fifth way

It is possible to construct the midpoint of a given segment only by means of a ruler in case a straight line l is given in the plane parallel to the segment.

The construction is carried out as in the preceding way but here the parallel straight line constructed with the bilateral ruler is given. Therefore, the remaining structures can be executed only by means of a ruler.



Drawing 6

Thirteen solutions of this basic problem for construction and four applications of this problem are given in [4]. In this case, it is not the number of solutions that is important; important is the fact that the process of obtaining of these solutions can be managed.

CONCLUSION

It becomes obvious from the above said that the formation of skills for searching for solutions of a problem (carried out as a concrete activity) is manageable and can be supported in the case when the theoretical knowledge and the methods of action adequate to them are known and systematized in a suitable way and if sufficient exercises for their learning and support have been done. If any of the solutions is looked for so that to be based on a certain theory or on its respective method of action, then the solution might not be „rational". Finally, the result is important and it is that the solution has been found on the basis of the practiced theory, which explains the construction found.

REFERENCES

1. Ganchev, I., Essential activities in a mathematics lesson, Modul-96, Sofia, 1999. (in Bulgarian)
2. Granovskaya, R. Elements of practical psychology, Saint Petersburg, 2007. (in Russian)
3. Landa, L. N., Algorithmization in learning and instruction, Prosveshtenie, Moscow, 1966. (in Russian)
4. Ninova, Yu., Essential construction problems, Makros, Sofia, 2007 (in Bulgarian)
5. Gantchev Iv. Concepts Representing (Fixing) Mathematical Activities And Their Place in Logic And Didactics of Mathematics, Current Trends in Mathematics Education, 5th MEDCONF 2007, Rhodes, Greece.