CERTAIN ELEMENTS OF GEOMETRY IN GREECE AND CYPRUS IN THE 19TH CENTURY. A DETAILED REFERENCE ON CENTER OF GRAVITY AND TRIANGLES SIMILARITY FOUNDED IN KORES GEOMETRY TEXT BOOK DATED 1903

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ABSTRACT

The history of the teaching of Mathematics has become one of the research areas into the Didactics of Mathematics over the past decades. In particular, the study of the history of teaching Euclidean Geometry offers valuable information for the redetermination of its course within maths education as a whole This work represents a first attempt at researching this topic within a Cypriot framework. In the first part, the History of Education within Cyprus in 19th century is examined, providing facts about the educational system, the analytical programs, teaching methodology and books or other documents used. In the second part, a brief comparison is made of the content and the philosophy that lies in three Geometry books used at the Pancyprian Gymnasium during the time in question. An extensive reference on the centre of gravity and triangles similarity is then discussed, which was founded only in Kore' s Geometry book 1903.

THE HISTORY OF MATHEMATICAL EDUCATION AS A RESEARCH TOPIC INTO THE DIDACTICS OF MATHEMATICS

"According to Geoffry Howson, the history of Mathematics teaching is the workshop of one who develops study programs. By studying the subject, we better understand its complexity and the way that Maths education interacts with society" (Shubring, 1993, p. 25).
Gert Schubring (Shubring, 1993, σ.29) cites that traditional approaches are restricted to administration document analysis, such as programs or circulars and to

the study of text books. Administration document study is carried out without the examination of the complexicity of the system that defines the realistic school education. On the contrary, the study starts with the conception that a teaching book speaks for itself whilst a historical text can only be understood in context. The problem that arises here is the analysis of the relationship between the text and its setting. Education and school are two social activities, so it is quite obvious that we should seek the methodology within a framework of social history.

On the basis of the above Gert Shubring suggests a look at the history of mentalities. This was applied by the well-known school of Annales in France. The main axes of this methodology are the theory of social history and the transformation of social knowledge. Social knowledge refers to an existence of strata or social groups, which support knowledge. The basic sections of this program are: (a) knowledge body, (b) social strata which support it, (c) means of dissemination and transformation, (d) results. Such a view gives the possibility of compromising two opposite poles: the object (knowledge) and its modification in the human mind as a social act.

With the above criteria, the analysis of Geometry books of the 19th century and early 20th century for Greece and Cyprus includes:

a) Mathematical scientific knowledge: where it was at the start and how it developed over this period;

b) Didactical transformation: for secondary education teaching, part of which also appears in teaching books;

c) Analytical programs: with teaching periods;

d) Historical and social framework: of this period;

e) Biographies: in order to analyze professional life, education, origin, career, professional activities in addition to teachers' ideology of that era;

g) chronological and ideological differences between geometry textbooks, from author to author.

This project is divided in two parts. In the first part, we study Mathematics education in Cyprus in the 19th century, with references to the educational system, the analytical programs for Geometry, the didactic methodology and teaching books used. The second part is a general comparison of three Geometry books used both in Greece and in Cyprus. This involves editions of three different authors, with chronological publications for the years 1874, 1878 and 1903 and a special reference on centre of gravity and shapes similarity founded in Kores book.

MATHEMATICS EDUCATION IN CYPRUS

According to historical sources, the educational system in Cyprus was in accordance with that of Greece. This is not surprising considering the fact that Cyprus was settled by Mycenaean in 1600 BC and as a result the native Cypriots incorporated and adopted the Greek civilization and its values. Since then, Cyprus has run a parallel course with Greece, closely following the same historical and civilized route, even today. Cyprus has been a vehicle of Greek-Christian civilization since Byzantine times. Their occupation by Venetians and Franks congregated Cypriots around the Orthodox Church, which naturally took on a protective role. The need for learning to read religious texts led bishoprics and monasteries to the foundation and maintenance of schools all over the island. The situation continued during Turkish Occupation too, where the need for Greek education became even more intense.

HISTORY

We examine the history of Cyprus secondary school education by observing the history of schools in Nicosia where there are available written sources. The reason we use Nicosia schools is because the other towns followed some time later.

In 1741, Archbishop Filotheos founded the "School of Greek Education and Music" («Σχολή των Ελληνικών Γραμμάτων και Μουσικής»). "Efrem o Athineos" (Efrem from Athens), the well-known Greek master, taught at the school following the program of School of Patmos ("Πατμιάδα Σχολή"). Here the program included: the alphabet, basic religious books and Byzantine Music. The books were chosen by the Archbishop and the school headmaster. The school's main objective was to prepare teachers for other schools which existed in other towns and villages, by teaching them the standard interteaching system of education (Περιστιάνης, 2000).

This first school of this kind closed in 1808 for unknown reasons. In its place the "Greek School" («Ελληνική Σχολή») was founded in 1812 by the national martyr Archbishop Kyprianos, with the aim to educate the people of Cyprus using the Greek language. The school closed after 9th of July 1821 and was followed by the opening of the "Central School of Greek Education" («Κεντρικόν Σχολείον της Ελληνικής Παιδείας») founded by Archbishop Panaretos. This school had branches in Larnaca and Limassol. The educational program was almost exclusively limited to religious education and reading and writing using the interteaching system (Σπυριδάκις, 1944).

The first regulations of the school were recorded in 1859 where it plainly mentioned the teaching of "Religious Studies, Greek, Mathematics and as foreign languages Turkish, French and Italian." The school continued to operate smoothly, adapting its programs year after year to correspond with those of Greece. According to newspapers of that time, in 1887, the school had five (5) classes (that is two less than any Greek Gymnasium), however, its graduates could enter as students in to the final year in a Greek Gymnasium (Σπυριδάκις, 1944, p.6).

Cypriots and fellow country men living overseas, demanded the foundation of a Gymnasium similar to the Greek ones, with a fully recognized Apolytirion (school leaving certificate). This would enable Cypriots to further their studies in Greece rather than Syria. At the same time (1892), a law was passed, preventing anyone who did not possess an apolytirion from one of the "Greek Schools" in Nicosia, Larnaca or Limassol from being appointed as a teacher. Meanwhile, the Cypriot Fraternity in Egypt decide to financially aid the foundation of a Gymnasium having equality with the ones in Greece. On 16th May 1893 Archbishop Sofronios calls a citizens meeting in Nicosia where it is finally decided to create such a Gymnasium.

THE PANCYPRIAN GYMNASIUM (ΤΟ ΠΑΓΚΎΠΡΙΟ ΓΥΜΝΆΣΙΟ)

The foundation committee of the Gymnasium applied to the Athenian Philological Association Parnassos requesting: a headmaster who had graduated in Teaching from a university in Europe, a Greek teacher specialized in Latin, a Theologian who was also a skilful preacher and a teacher of Maths and Physics. The Associations president, Nikolaos Politis, sent competent teachers for the above posts, amongst them a pioneer in teaching, the Maths and Physics teacher Nicolaos Katalanos.

From 1983 up to now the Pancyprian Gymnasium is considered to be the highest intellectual institute of Cyprus. Parallel to its educational role, it has developed cultural, spiritual and national action. Every year the school drew up its program according to that of the equivalent "Classic Greek School" and submitted it to the Greek Ministry of Education for approval. Therefore, we have a completely identical school program for Cypriot and Greek Gymnasiums. Almost all schools founded afterwards in villages and towns until 1960 were of the same kind (Περσιάνη, 1994).

At the inauguration of the "Pancyprian Gymnasium", 12th December 1893, in his speech, the head teacher Dellios, talks about the studies program and the philosophy of education in general. The studies program includes: (a) ancient Greek language, Literature, History and Latin, (b) Physics-Mathematics, (c) foreign languages (English and French) and (d) practical lessons, sports music and art. As far as the philosophy of education is concerned, its main consideration is the education of people for moral and national training and refinement, but not for professional training. The content of the lessons is sufficient for the achievement of the set goals whereas the teaching methods are different. Finally, the development communication and mutual understanding with more advanced European nations, also is an important parameter. The general idea is that the school offers education to future leaders of the Cypriot community (Περσιάνης, 1994, pp.31-32).

EDUCATION SYSTEM

Up to year 1892 a uniform education system did not exist. With the foundation of the "Pancyprian Gymnasium", three basic educational objectives are set for the educational policy. First, that the school provides proper Greek education, second, to create a school identical to those in Greece and third, the recognition of this school by the Greek government.

In spite of this, a new structural type school appears- the seven (7) class gymnasium, accepting pupils from a four (4) class primary school. Here, 4 gymnasium classes functioned as well as 3 classes lower in level than Greek schools. The program offered optional Pedagogical teaching to sixth- formers- preparing them as potential teachers. The headmaster was appointed and supervised all Nicosia primary schools also being responsible for the planning of the teaching program. Between 1896- 1911 the school is modified into a six class gymnasium based on the fact that now primary schools had developed into six year attendance. This (6 year primary and 6 year gymnasium) system later moved across to Greece where it was maintained for many years (Σπυριδάκις, 1944, p.20).

GEOMETRY TEXTBOOKS

The Pancypria Gymnasium naturally uses textbooks approved by the Greek Ministry of Education. Between 1882-1892 *"the only approved textbooks"* were the series published by Ioannis Hadjidakis. Therefore, in A. Theodorou's order form for textbooks (28th August 1894) we find 3 of Hadjidakis books: "Fundamental Trigonometry", "Algebra" and " Theoretical Arithmetic". The same titles were repeatedly ordered up to 1899. In the "Severios Library" (on the school premises) and the Gymnasium archives we find other textbooks. In total eight (8) titles were found to belong to the examined period.

DIDACTICAL METHODOLOGY

We are uncertain of the teaching methods which were used especially in Mathematics. Nikolaos Katalanos, mentions in his "Memoirs", an innovative method he used which aroused the interest, not only of his students, but of long standing teachers too. On this basis we can assume that Katalanos introduced the teaching method of Herbard in Cyprus, almost simultaneously with Greece. In any case, the interteaching method was maintained in Cyprus for many years to come due to lack of teachers especially for primary schools (Περιστιάνης, 2000).

ANALYTICAL PROGRAMS

Dellios, in his speech 12th December 1893, satisfactorily described the cognitive and emotional goals for Physics and Mathematics and for other lessons on a whole. For Physics and Mathematics he states:

"The cognitive objectives are to make students aware of the 'living and lifeless nature that surrounds them', to understand 'the abstract relations (of natural objects) with aspect to the number, its size and shape', to acquire 'positive empirical knowledge and an unplacable sense of objects' and to protect themselves 'from being dangerously misled' and to awaken their spirit to 'move from relative to absolute and to think by means of general and abstract concepts, a feature exclusive to humans'. The emotional objectives are to rid students of superstitions and to help them develop into 'intellectually free and independent people'" (Περσιάνη, 1994, p. 30).

In the analytical program of 1903 it is mentioned, in the letter for approval to Greek Ministry of Education, that this program has been in use for the past six years. From the timetable it is observed that out of 217 periods, 96 relate to

Philological (Literature) lessons and 32 periods to Physics and Mathematics lessons. More specifically, the Maths teaching program states:

> **1st form:** Practical Arithmetic: writing and recitation of numbers, divisibility, fractions, decimals, mixed numbers, proportional and inversely proportional amounts and simple and compound method of three.
>
> **2nd form:** Practical Arithmetic: simple and compound methods of interest problems, averages, powers and square roots. Theoretical Arithmetic: for counting, for four operations of integers.
>
> **3rd form:** Theoretical Arithmetic: divisibility, fractions, decimals, mixed numbers, square roots, ratio and proportions. Geometry: Γεωμετρία: the first two books of plane geometry.
>
> **4th form:** Geometry: the 3rd book of plane geometry *"with exercises and ground applications"* Algebra: introduction, algebraic calculations and first degree equations.
>
> **5th form:** Geometry: the fourth book of plane geometry and the first two of solid geometry. Algebra: irrational numbers, roots and powers, second degree equations, progressions, logarithms and compound interest.
>
> **6th form:** Geometry: the last book with exercises and applications on a whole. Straight line Trigonometry.

PRESENTATION AND COMPARISON OF TEXTBOOKS

The Pancypria Gymnasium naturally uses textbooks approved by the Greek Ministry of Education. Between 1882-1892 *"the only approved textbooks"* were the series published by Ioannis Hadjidakis. Therefore, in A. Theodorou's order form for textbooks (28th August 1894) we find 3 of Hadjidakis books: "Fundamental Trigonometry", "Algebra" and " Theoretical Arithmetic". The same titles were repeatedly ordered up to 1899. In the "Severios Library" (on the school premises) and the Gymnasium archives we find other textbooks. In total eight (8) titles were found to belong to the examined period.

In an attempt to research the changes that appeared in Geometry textbooks for both Greece and Cyprus for the examined period, we primarily concentrated on what has been indicated to have been used in Cyprus. Turning towards the Severios library and the Gymnasium archives, we found a series of 8 textbooks for the period 1874-1935. From these, we chose 3 main titles which we had access to. Our study pinpoint the differences between their content and philosophy. These books are:

1. Demetriadis G. (1874). "Elements of Geometry" («Στοιχεία Γεωμετρίας», Theory, Practice and Applications, for schools. Part A: Plane Geometry. Constantinople: Voutyras and co.

2. Damaskinos A. (1878). "Elements of Geometry, Legendre" («Στοιχεία Γεωμετρίας Λεγένδρου») modifications and additions. Fifth elaborated edition. Athens: I. Aggelopoulos Publishers.

3. Kores M. (1903). "Elements of Geometry" («Στοιχεία Γεωμετρίας») for gymnasium student use and preparation school use. Athens: Estia.

GEOMETRY TEXTBOOKS, GENERAL CHARACTERISTICS

Antonios Damaskinos is a professor with strong French influence. His book is a translation of Legendre's elements of Geometry. Legendre's Geometry dominated the teaching of geometry in Greece during the 19th century (Γαγάτσης, 1993). In his prologue, he presents his views on the role of schools and teachers as well as the value of a textbook. Important at the time is the great attention he pays to clarity of proofs and the language of the texts, thereby recognising (maybe not intentionally) the role these two elements play in the comprehensibility of the texts and consequently in students understanding (Γαγάτσης, 1993). The content is divided into Plane- Geometry and Solid- Geometry, and includes 8 books (chapters) with the same structure used by Legendre.

George A. Demetriades, a mechanic, directs his book to students and others who wish to acquire practical knowledge of Geometry and Topography. Thus, in each paragraph he gives practical applications with problems from Land Surveying and Mechanics. In the introduction of his book, he makes extensive reference to the history of Mathematics covering Ancient Greece, Byzantium and Europe with Descartes, Newton, Leibnitz and Laplace. He also gives a brief account of Greek mathematicians. The author argues that the existing textbooks are completely theoretical and as a result the concepts are misunderstood by maths students. His book is only concerned with Plane Geometry and it is divided into 13 paragraphs. Instead of a title, the heading gives a summary reference of the content of each paragraph. In this way, in the first paragraph we find:

> *"Points and straight lines on the earth or on a map. Plane, Definition of angles. Applications. Drawing straight lines on the earth or maps. Measurements of lines, Chain, Pole, Tape measures, Steel Decameter, Vernier"* (Δημητριάδης, 1874, p. 5).

The content structure generally follows Legendre's structuring but its parts are more detailed.

M. Kores, is a Mathematics lecturer. His book addresses Gymnasium and Preparatory school students. During this period, each student has for the first time his own textbook. Previously, textbooks were only for teachers use. Kores marks with an asterix the parts of Geometry which may be omitted, due to lack of teaching time, a common practice at this period. However, he points out that these parts are essential for students who wish to pursue Mathematics in the future.

In Kores book we encounter the standard separation of Plane Geometry and Solid Geometry and a similar structure with that of the other authors. The edition presents problems in the numbering of chapters and sub- paragraphs with the unanswered question, of course, whether the errors are an oversight of the author or the publisher.

The format of the three books has many advantages. The development of printers at this time allowed the insertion of figures into the texts. Kores book displays the best printing although Demetriades book shows impressive typographical work. The sophisticated wood- engravings support the structure of the theory, emphasizes the several meanings and the aesthetics of the book.
A more detailed comparison of the above-mentioned textbooks can be found at our paper ‘‘*Certain Elements of Geometry in Greece and Cyprus of the 19th Century*’’, 4th Mediterranean Conference on Mathematics Education, Palermo, Italy, January 2005.

AN EXCEPTIONAL THEOREM

At the end of the chapter on Polygons Kores gives the following statement:

"Given various points on a plane, a point can always be found on it, whose distance from a straight line lying on the same plane and taken as an axis, is equal to the mean of distances of all the points from this axis" (Kores, 1903, p.47).

The proof of this statement is achieved by taking successive polygonal lines, which have as convexes the means of the previous lines. Thus, using a marginal approach we find the point that is called "centre of medium distance of initial points". From the two special cases mentioned by Kores:

"In any given triangle the centre of mid distances of the convexes coincides with the intersection point of the median lines",

"... in any given quadrilateral, the center of mid distances, coincides with the midpoints of the opposite conjunction lines " (Kores, 1903, p.49),

it becomes clear that the term “center of mid distances” is another name given to the center of gravity of a shape. According to Gkioka, Carnot introduced the term center of gravity of a triangle instead of center of mid distances. He, also, reports

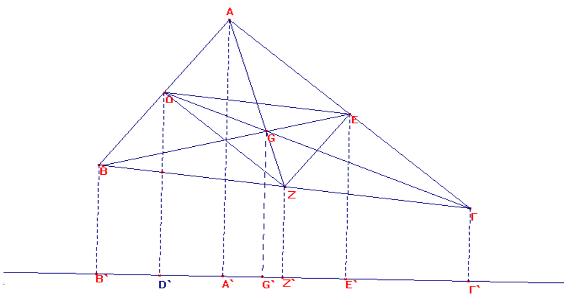
"we believe that the introduction of the term center of mid distances or center of gravity in the” Elements of Geometry” was made by Bobillier, in the Cours de Geom. p.55-83" (Gkiokas, 1952).

We can find a similar statement to the one of the triangle in the " Geometry Exercises of Jesuits " written by Gkioka as follows:

"The sum of the distances of the convexes of a triangle ΑΒΓ from any straight line μν is equal to the sum of the distances of the midpoints of its sides from that straight line"

Proof: $\Delta\Delta'=(BB'+AA')/2$ and $ZZ'=(BB'+\Gamma\Gamma')/2$,

$EE'=(AA'+\Gamma\Gamma')/2$



Therefore,

ΔΔ´+ZZ´+EE´=AA´+BB´+ΓΓ´

Note:

Working in a similar on the triangle ΔEZ, etc, we will find the point of intersection of the medians G. Hence, 3GG`=AA`+BB`+ΓΓ` .

Despite the absence of this statement from other Geometries, some special cases appear in the form of exercises.

In the book of Petros Togka "*Exercises and Problems of Geometry*" (publication 16th), we found three such exercises. The first of them ,number 272, is as follows:

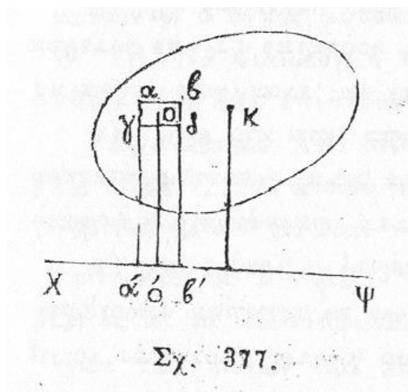
"The sum of the distances of the convexes of a parallelogram from a straight line which does not intersect the parallelogram, equals four times the distance of the intersection of the diagonals of the parallelogram from that straight line" (p.115).

The other two exercises, numbers 291 and 292 (p.125-126) concern the triangle. For this three exercises Togkas proposes the use of the Jesuits' solutions with the trapezoid method.

The above theorem does not appear in Damaskinos, Dimitriadis or any of the instructive books of Geometry that we studied, i.e. the "Theoretical Geometry of" P. Tonga, the "Big Geometry of" N. Nikolaos (1962) and the modern Euclidean Geometry of Thomaidis, Xenou, Pantelidi, Poulou, Stamou (2000) and Argiropoulou, Blamou, Katsouli, Markati Sideri (2000). It does not exist in the "Elements" (K.E.EP.EK., 2001).

A relevant theorem, a special case of the one above, appears in «Euclidean Geometry» of Spyros Kanellos (1975), which states:

«*When drawing any straight line passing through the intersection point of the medians of a triangle ABC, the sum of the distance of the line from the two convexes of the trangle lying on the same side of the line equals the distance of the line from the third convex*».



The above theorem helps in proving the theorem referring to the revolution of a triangle around any axis which states:

«*The volume produced by the revolution of a triangle around an axis belonging on the same plane as the triangle, having no common points with the triangle, is equal to the product of the area of the triangle multiplied by the circumference drawn by the centre of gravity of the triangle when revolved*».

The generalized version of this proposition is the famous theorem of Papous of Alexandria, applying not only on triangle or polygons but also to any closed shape of the plane. The theorem, according to D. Tsimpouraki, appears in the eighth book of Papous. It applies to the theory of Mechanics, more specifically the theory of centres of gravity and simple machines, where the study is strictly based on the criteria of Euclidean Geometry on natural solids (Tsimpourakis, 1985, p 392-393).

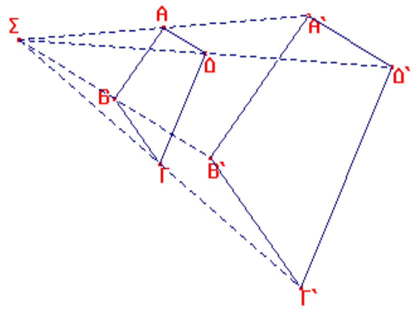
Kores includes Papous Theorem, without any specific reference to it, in the last part of his book. The 618 theorem states:

«*The volume produced by any closed area of a plane, revolving about an axis of the same plane, is equal to the product of the generating area and the circumference drawn by the centre of gravity of the area*». (Kores, 1903, p 390).

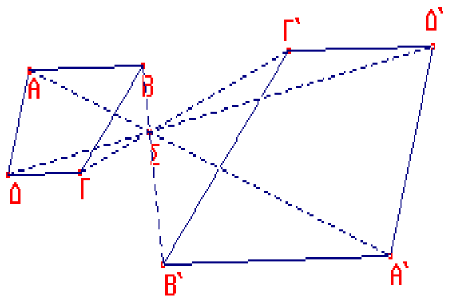
The proof of this theorem is achieved using the “infinite points of the given area” which are created by drawing horizontal and vertical parallel lines, a logic which can be found in calculus. The theorem appears right after the theorem referring to the area produced by the revolution of a straight line. This theorem is proved using “infinite equal parts” of the line.

In Kores, the chapter on similarity, takes great importance due to his reference to homothesis. The homothesis is introduced as order of two shapes and not as points transformation. Therefore, two polygons are congruent if they are homothetic equal and have their corresponding sides parallel. The straight lines connecting the corresponding convexes are called “carrying radii” and their common point Σ, is called the centre of homothesis.

The difference between direct homothesis and inverse homothesis is in the position of the two polygons. In direct homothesis both polygons lie on the same side of the centre of homothesis, whereas in inverse homothesis the polygons lie in opposite sides of the centre.



Direct homothesis

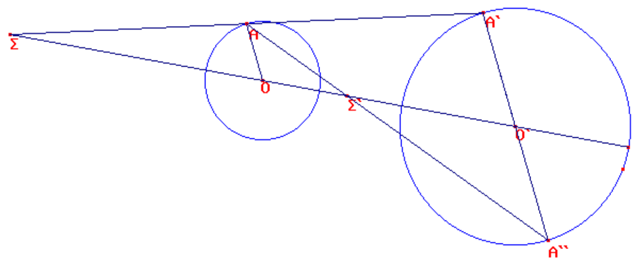


Inverse homothesis

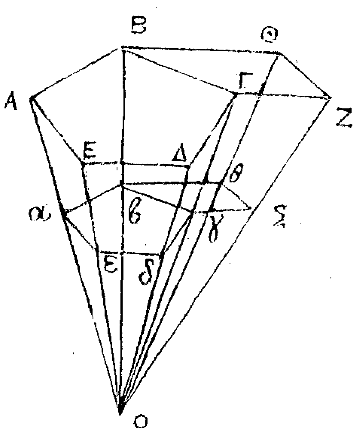
In both cases the ratio ΣA/ΣA´ (A and A´are the Corresponding convexes) is called “similarity ratio” (Kores, 1903, p 108-110).

Looking at the diagrams of the two homothesis cases, it is easily understood, as the author notes, that the two inversely homothetic polygons can be converted to

directly homothetic by rotating one of them 180° about the centre of homothesis. Finally, he suggests a method of finding all the homothetic polygons of a given polygon, by changing the ratio of homothesis from zero to infinite.
The theorem of homothesis of two circumferences is of great interest, as well. It states:
«*Any two circumferences are at the same time directly and inversely homothetic*». (Kores, 1903, p. 111)



The proof is based on the fact that the two radii OA and O΄A΄ can be parallel either when drawn in the same direction or when drawn in opposite directions, having a constant ratio. In the diagram can be seen the way that the two centres of homothesis Σ and Σ΄ are created. In a conclusion note the writer state for the homothesis of two circumferences: «…*the common tangents having on the same side the two circles (external) passes by the centre of direct homothesis, whereas the two common tangents having the two circles on opposite sides pass through the centre of inverse homothesis of the two circles*» (Kores 1903, p.112). We have to mention, for historical reasons, that according to the Jesuits the study of congruent shapes is introduced and examined by Thales. On the other hand the direct omothesis spapes (direct situated) were examined by Poncelt 1822 (Traite des proprietes projectives des figures, tome I chap, III). The term ¨*homothetical shapes*¨ was given by Chasles 1827 (An. D. Georgonne, tome XVIII, p. 280) as well as the term ¨ *congruent centre* ¨, the full study was done by Euler (Gkiokas, 1952, p.492). Kores returns to homothesis at the end of his book at chapter «*On homothetical solid shapes*» (Kores, 1903, p.359), where he expand homothesis in three dimensions, giving the proofs of all the statements starting with the «*if we connect any point O with the convexes of a given polyhedron Π and take on the directional radii , OA, OB, OG,… the lengths Oa, Ob, Oc, .. in such a way that we have equal ratios*



$$\frac{OA}{O\alpha}=\frac{OB}{O\beta}=\frac{O\Gamma}{O\gamma}$$

the polyhedron Π having convexes the points a, b, c is similar to the polyhedron Π» (Kores, M., 1903, p359).

Moving on to the proof of the theorem, point O, is called, using the necessary justification, centre of line or/ and of inverse similarity. Afterwards, he defines homothesis as: «*Briefly, we define homothesis the similarity in shape and position, of solids we have already studied above. These polyhedra are called homothetic, the point O is the centre of the direct or inverse homothesis and the constant ratio* $\frac{OA}{O\alpha}$ *is the ratio of similarity or homothesis*.» (Kores, M., 1903, p 360). Next, it is clarified that the definition of homothesis is the same for solids as well as two dimensional shapes. The homothesis of points, lines, planes, circumferences, spheres, cones and cylinders is examined.

The introduction of homothesis in Geometry enforces the conclusion that Kores was influenced by Analysis.

Referring to homothesis in such a simplified way, he has in mind the geometric transformations.

CONCLUSIONS

- A French influence in the didactics of Geometry, many translations of Legendre's
  book from the 19th up to the mid 20th century.
- Greece and Cyprus use the same didactical Geometry text books.
- Main didactic method is the interteaching method.
- The Herbard method is introduced to Greece 10 years earlier than Cyprus.
- The teachers (of the Greek School and Gymnasium) are graduates from Greek, French and German universities.
- There are no important differences in the analytical programs between both countries.
- The three authors do not follow the "Elements" of Euclides.
- Damaskinos: Strong French influence. His book is a translation of Legendre's. Marked use of intuitive rules and proofs. General course from general to specific.
- Demetriades: A desire to adapt to the production need. He does away with "unnecessary" statements but has a wealth of simple proofs and many applications. He defines special quadrilaterals as separate entities..
- Kores: Incorporates elements of Vector and Calculus Analysis. An attempt to introduce modern ways of thinking. He defines lines in terms of moving points and surface planes in terms of moving lines. Defines the circle as a locus of point.
- Kores is giving the distinct theorem of "*the centre of average distances*", not founded in other Greek Geometry textbook.
- He uses homothecy of planes and space and examines similarity in a complete different way than Euclides. Within a series of proofs he uses the limits and infinitesimal.

GENERAL CONCLUSION

- Even today there is an attempt to find a balance between the historical legacy of Euclides and new mathematical ideas.
- A study of the history of the Didactics of Geometry and the different perceptions noted in textbooks is probably the best way to enrich our knowledge, not just in mathematics but for didactic transformation.
- The area of research of this project opens itself up to further study.

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