MOTIVATION OF STUDENTS FOR RESEARCH WORK USING ROTATION BODIES GEOMETRY

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ABSTRACT

The study considers the types of rotation bodies received when a triangle is rotated around a line in the same plane in space. A few cases can be classified: the triangle and the line have zero, one, two or countless common points. In each of these cases the line can be perpendicular or parallel to a side of the triangle or can be in a general position. We show how to stimulate naturally the exploration of the problem or of its elements when working with schoolchildren. The research has been done taking advantage of information technologies and there is a tailor-made video clip which can be seen on the Internet at http://www.youtube.com/watch?v=-WZmrvVDbn0

When schoolchildren study rotation figures in maths classes, they also solve problems in which a surface figure rotates around a straight line in the same plane. At the end of that section of the mathematical problems book, the teacher could offer the 12-grade students to review the type of problems that they have already solved, as well as to make a classification and generalization. This is followed by a review of the problems in the textbook and in some mathematical problems collections. The students come to the conclusion that the figures rotating around a line are mainly triangles and some quadrilaterals. The problems that predominate are those in which a triangle rotates around a side, but there are also problems in which the line goes through an apex of the triangle or it either doesn't cross the triangle or the triangle rotates around a height. There are all kinds of triangles depending on the angles. Thus, there logically arises the question about the type of figures received in space when a triangle rotates around a line depending on the kind of the triangle and the position of the line in the same plane. Schoolchildren could be offered to do some research in this direction. Since this task is difficult, large, time-consuming and requiring a lot of efforts, it can be done by the students interested in maths after their classes. The research can be divided into parts, observed by different groups of students. The teacher should give instructions on how to perform the task in order to consider all interesting cases and to avoid omitting any of them. The mechanism that could be applied is the rotation of the triangle around an apex (e.g. B) in the plane and the search for a result in each position of its rotation around an immobile straight line (Fig. 1). A pattern of different triangles can be used, but it will be much better to apply computer technologies. Finally, the results are systematized, supplemented and presented to all students. The results can be classified into the following cases: the triangle and the line have no common points, or one common point, or two common points or countless common points. In the course of the research, when there is a change in the reciprocal position of a triangle and a line, it is seen that there are different cases when the line is perpendicular to a side of the triangle; when the line is parallel to a side of the triangle and when there is a general position different from the ones that have already been mentioned.

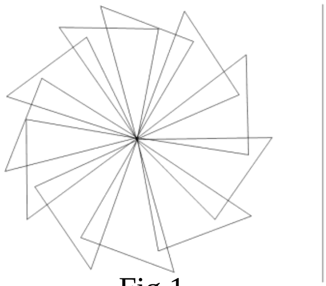

Fig 1

I. THE TRIANGLE AND THE LINE HAVE NO COMMON POINTS

1. Line g is perpendicular to a side of the triangle.

In the case of acute-angled and obtuse-angled triangles the rotation body obtained is a truncated cone in which there is a hollow with the form of a truncated cone. (Fig. 2).

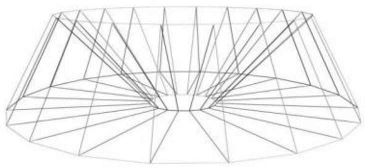

Fig 2

The same is the result when there is a right-angled triangle and the line is perpendicular to its hypothenuse. When line g is perpendicular to one of the sides of the right triangle, the result is a truncated cone with a cylindrical hollow (Fig. 3) or a cylinder with a hollow having the form of a truncated cone (Fig. 4).

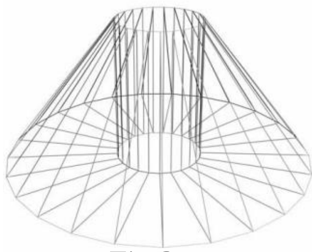

Fig 3

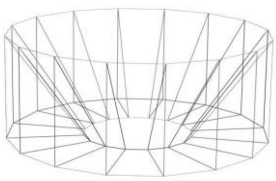

Fig 4

2. Line g is parallel to a side of the triangle.

In the case of the acute-angled triangle the rotation figure is a combination of two truncated cones in which there is a cylindrical hollow (Fig. 5) or a cylinder in which there is a hollow consisting of two truncated cones (Fig. 6).

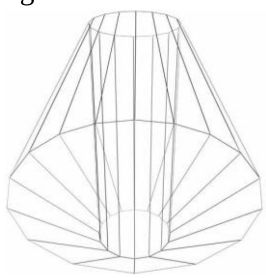

Fig 5

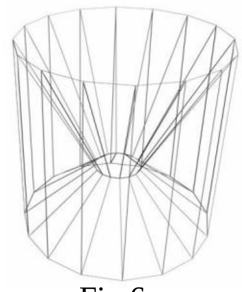

Fig 6

In the case of the obtuse-angled triangle, if g is parallel to the side lying opposite the obtuse angle, the body obtained is again the combination of two truncated cones in which there is a cylindrical hollow. If g is parallel to one of the smaller sides, the result from the rotation is a truncated cone with a hollow which is a combination of a cylinder and a truncated cone (Fig. 7) or a combination of a cylinder and a truncated cone with a hollow having the form of a truncated cone (Fig. 8).

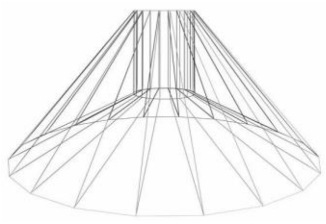

Fig 7

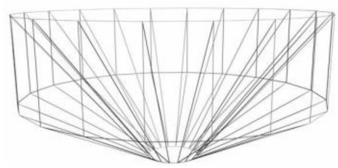

Fig 8

When we rotate a right triangle around a line parallel to the hypothenuse, the result is a combination of two truncated cones in which there is a cylindrical hollow. This case is similar to the case of the obtuse-angled triangle when g is parallel to the side lying opposite the obtuse angle. If g is parallel to a side of a right triangle, then the result is a truncated cone with a cylindrical hollow or a cylinder with a hollow having the form of a truncated cone. This case does not differ from the case in which g is perpendicular to a side of a right triangle.

3. Line g is in a general position.

With all types of triangles- acute-angled, obtuse-angled and right ones, we have the same result from the rotation around straight line g – either a truncated cone with a hollow consisting of two truncated cones (Fig. 9) or a combination of two truncated cones in which there is a hollow having the form of a truncated cone. (Fig. 10).

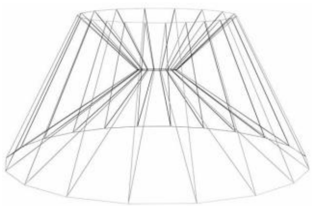

Fig 9

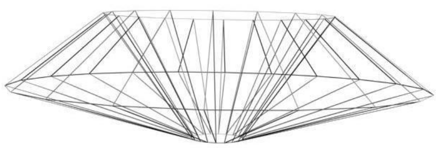

Fig 10

II. THE TRIANGLE AND THE LINE HAVE ONE COMMON POINT

1. Line g is perpendicular to a side of the triangle.

In the case of the acute-angled triangle, the result from the rotation is a truncated cone with a conic hollow (Fig. 11).

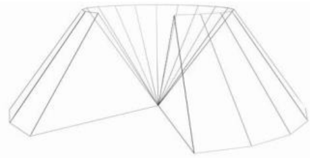

Fig 11

In the case of the obtuse-angled triangle, if g goes through the apex of an acute angle and is perpendicular to the longest side, then the received figure is the same as the figure received in the case of the acute-angled triangle. If g goes through the apex of an acute angle and is perpendicular to the opposite side, then the figure received with the rotation around g is a cone with a conic hollow (Fig. 12). If g goes through the apex of the obtuse angle, then g has two common points with the triangle no matter which side it is perpendicular to.

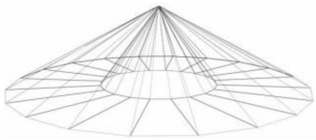

Fig 12

In the case of a right triangle, if g goes through the apex of the right angle, it has more than one common point with the triangle. If g goes through the apex of an acute angle and is perpendicular to the hypotenuse, the result is similar to the result in the case of the acute-angled triangle- it is a truncated cone with a conic hollow. If g is perpendicular to the side of the right triangle it crosses, the result from the rotation is a cylinder with a conic hollow (Fig. 13).

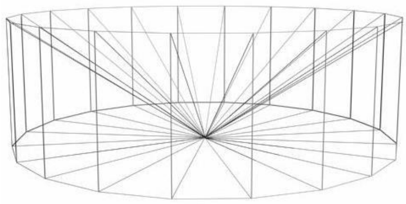

Fig 13

2. Line g is parallel to a side of the triangle.

When we rotate an acute-angled triangle, the result is a cylinder with a hollow consisting of two cones. (Fig. 14). If g goes through the apex of an isosceles triangle, the cones which the hollow consists of are the same.
We have the same result when we rotate an obtuse-angled triangle if line g goes through the apex of the obtuse angle. If g goes through the apex of an acute angle, the rotation body consists of a cylinder and a cone in which there is a conic hollow (Fig. 15).

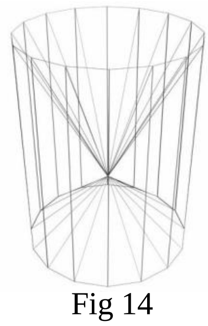

Fig 14

If we rotate a right-angled triangle around line g going through the apex of the right angle and is parallel to the hypotenuse, the result is analogous to that of the case of rotation of an acute-angled triangle- it is a cylinder with a hollow consisting of two cones. If line g goes through the apex of an acute angle and is parallel to the opposite side of the right triangle, we receive a cylinder with a conic hollow. This case is identical to the case in which g goes through the apex of an acute angle and is perpendicular to a cathetus.

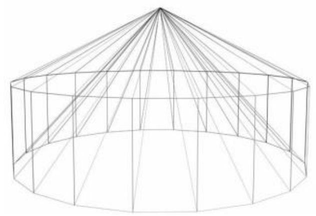
Fig 15

3. Straight line g is in a general position.

When we rotate acute-angled and obtuse-angled triangles around line g, we get surface of a cone and a truncated cone with a conic hollow (Fig. 16) or a truncated cone with a hollow consisting of two cones (Fig. 17). When we rotate an obtuse-angled triangle, we get either a truncated cone with a hollow consisting of two cones, or a cone with a hollow consisting of a cone and a truncated cone (Fig. 18).

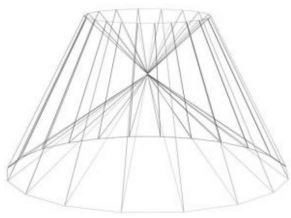
Fig 17

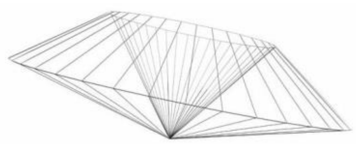
Fig 16

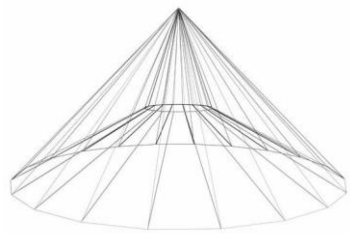
Fig 18

III. THE TRIANGLE AND THE LINE HAVE COUNTLESS COMMON POINTS

This is a case in which line g coincides with a side of the triangle. If the triangle is acute-angled, the figure received after the rotation is a combination of two cones (Fig. 19). Such is the result of the rotation of an obtuse-angled triangle when g coincides with its longest side and of the rotation of a right triangle when g coincides with the hypotenuse. If the observed triangle is obtuse-angled and g coincides with a smaller side, the result of the rotation is a cone with a conic hollow (Fig. 20). When we rotate a right triangle around a line coinciding with a side of the right triangle, the figure obtained is a cone.

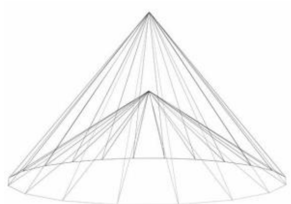
Fig 20

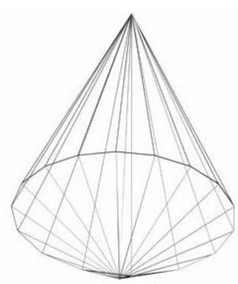
Fig 19

IV. THE TRIANGLE AND THE LINE HAVE TWO COMMON POINTS

The figures obtained when there is rotation of a triangle around a line with which it has two common points are the most interesting ones. They consist of complexly interwoven rotary figures and can be divided into two main patterns. The first one is when line g goes through an apex of the triangle and crosses the opposite side. The second one is when line g crosses two sides.

1. Line g passes through an apex of the triangle and crosses the opposite side.

With all kinds of triangles if g is perpendicular to the opposite side, the result from the rotation is a figure consisting of two inserted cones (Fig. 21). If the triangle is isosceles and g goes through the apex opposite the base, the inserted cones coincide, i.e. there is only one cone.

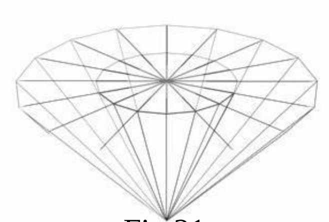

Fig 21

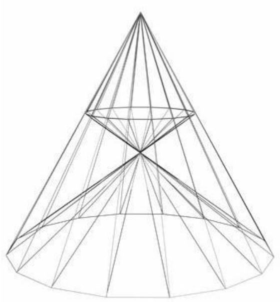

Fig 22

In the cases in which g goes through an apex of a triangle and crosses the opposite side without being perpendicular to it, the result from the rotation depends on whether the line is a bisector of the angle whose vertex it goes through. When g is a bisector, the result is a cone with a conic hollow but in the cone there are two more inserted cones with a common base. The circumference of this base lies on the outer cone (Fig. 22). When g is not a bisector, the result is a combination of three cones with a conic hollow. If in ΔABC AC>BC, g goes through point C and it is not a bisector, let us designate the point of intersection of the bisector through apex C with the opposite side AB of the triangle with L. Let g cross AB at point L_1 (Fig. 23). If L_1 is located between points L and B, then

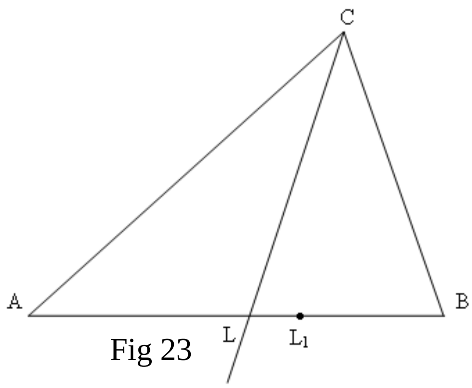

Fig 23

$$\frac{AL_1}{BL_1} = \frac{AL + LL_1}{BL - LL_1} > \frac{AL + LL_1}{BL} > \frac{AL}{BL} = \frac{AC}{BC}.$$

Fig 24

In this case two of the cones are completely inserted in the first cone (Fig. 24). This is how we come to the conclusion that if g goes through point C and crosses the opposite side in point L_1, located between points L and B, i.e. $\frac{AL_1}{BL_1} > \frac{AC}{BC}$, two of the cones are completely inserted in the first cone. If $\frac{AL_1}{BL_1} < \frac{AC}{BC}$, the two cones are not completely inserted in the first one but cut it (Fig. 25).

When AC<BC, two of the cones are completely inserted in the third cone if $\frac{AL_1}{BL_1} < \frac{AC}{BC}$; the two cones cut the third one when $\frac{AL_1}{BL_1} > \frac{AC}{BC}$.

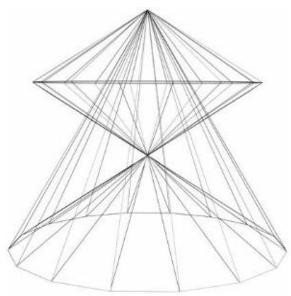

Fig 25

2. Line g crosses two sides of the triangle.

2.1. Line g crosses two sides of the triangle and is perpendicular to one of them.

Let us consider the acute-angled ΔABC, in which AC<BC (AC>BC – by analogy) and let line g be perpendicular to AB crossing it at point P. Let us designate the middle of AB with M (Fig. 26). If P is between points M and B, the result from the rotation of the triangle around line g is a truncated cone with an inserted cone and a conic hollow (Fig. 27). When g goes through point M (i.e. it is a perpendicular bisector), the bases of the cone and the inserted cone coincide. If P is located between points A and M, the result from the rotation is again a truncated cone with an inserted cone and a conic hollow. The same is the result when we consider a right triangle and g is perpendicular to the hypothenuse as well as when the triangle is obtuse-angled and g is perpendicular to the longest side.

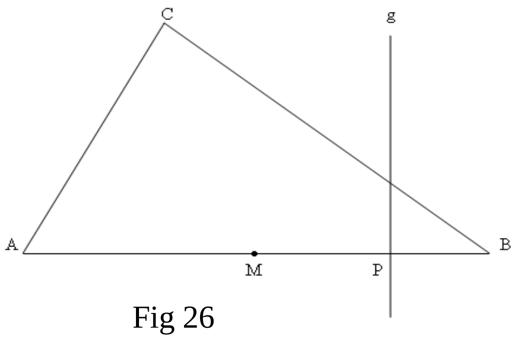

Fig 26

When ΔABC is right and if g is perpendicular to side AB, with P located between M and B, the result is a cylinder with an inserted cone and a conic hollow (Fig. 28). When g is the perpendicular bisector of the cathetus, the bases of the cylinder and the cone coincide (Fig. 29). If point P is between M and A, the result is a cylinder and a cone with a conic hollow but now the cone cuts the cylinder. (Fig. 30).

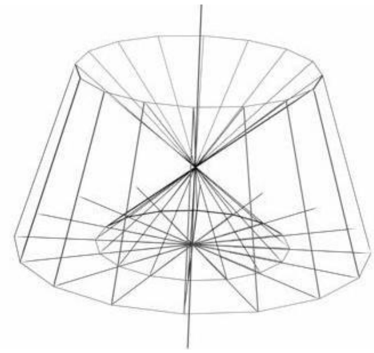

Fig 27

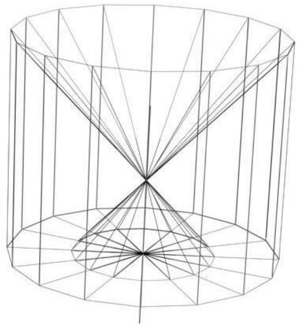

Fig 28

If ΔABC is obtuse-angled, let its obtuse angle be at apex B. Let us designate the middle of AB with M and let g be perpendicular to side AB, with g∩AB=P. If P is between A and M, the result from the rotation of the triangle around g is a truncated cone, an inserted cone and a conic hollow. With the movement of g towards point B the inserted cone grows and when g goes through point M, the inserted cone and the truncated cone have a common base with equal radii. When P is between M and B, the cone cuts the truncated cone. If g crosses the extension of AB, the result is a cone and two conic hollows, one of them cutting the cone (Fig. 31).

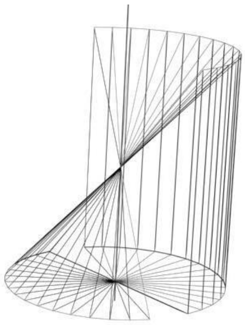

Fig 29

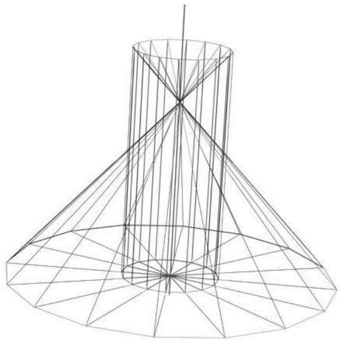

Fig 30

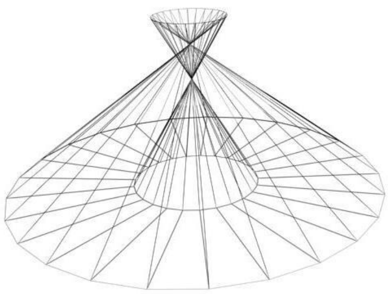

Fig 31

2.2. Line g crosses two sides of the triangle and is parallel to one of them.

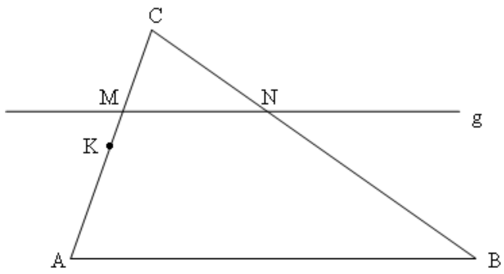

Fig 32

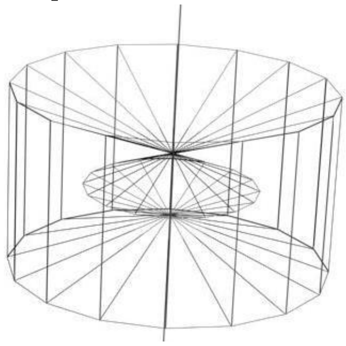

Fig 33

Let ΔABC be acute-angled and g∩AC = M, g ∩BC=N. Let us designate the middle of AC with K (Fig. 32). If M is between K and C, the result from the rotation around line g is a cylinder, two inserted cones and two conic hollows (Fig. 33). When M coincides with K, the circumferences which are bases of the cone lie on the surface of the cylinder, i.e. the radii of the cylinder and the cones are equal (Fig. 34). If point M is between A and K, the cylinder cuts the cones (Fig. 35).

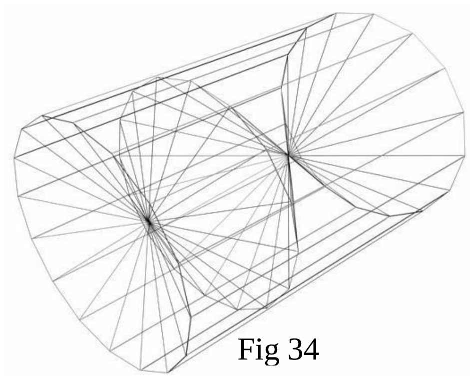

Fig 34

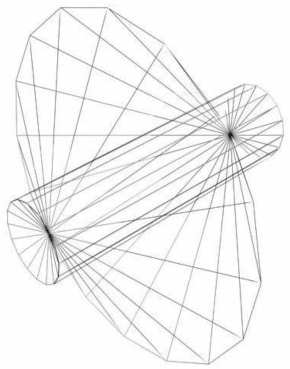

Fig 35

When ΔABC is right or obtuse-angled and g is parallel to the longest side, the result is the same as the result in the isosceles triangle case.

If ΔABC is right and g is parallel to a side of its, the case is the same as the one in which g is perpendicular to a side of the right triangle.

If ΔABC is obtuse-angled and g is parallel to a smaller side, the result from the rotation around line g is a cylinder, two cones and two conic hollows (Fig. 36).

2.3. Line g crosses two sides of the triangle and is in a general position.

The result from the rotation of the triangle around straight line g is a truncated cone, two cones and two conic hollows (Fig. 37).

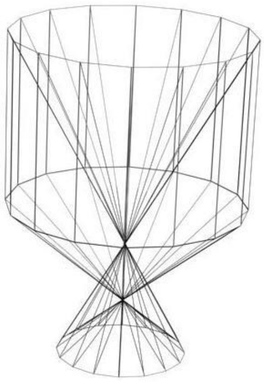

Fig 36

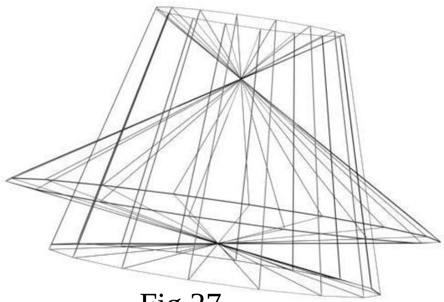

Fig 37

Generally, the students' research on rotation figures concentrates on the following cases: the triangle and line g have countless common points and the triangle and straight line g have one common point. The case in which the triangle and line g have two common points is not examined because it is the most complex one. However, it is the most interesting one. Applying computer technologies we can trace and analyse all cases. Thus, in the tailor-made animation clip at http://www.youtube.com/watch?v=-WZmrvVDbn0 we can see them in the process of movement of line g just as we can see the alteration of the type of triangle.