INDIVIDUAL SPECIFIC ABILITIES OF MOTIVATED STUDENTS IN PROBLEM SOLVING

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ABSTRACT

Individual specific abilities of motivated students in problem solving are investigated in the process of teaching problem solving. This paper explores the cognitive theory and its approach to learning. As this paper aims at research on personal motivation, the methods used are description and case study. Some observations are described and some conclusions are proposed.

I. THE PROBLEM

Motivation is the state which maintains, directs and encourages human action, and has a leading part in their behavior. Understanding what motivates people requires research on the direct and timely relation between the voluntary actions and the results they lead to. These results possess a subjective importance and are of psychological value for the individual.

Motivation is a crucial component for learning, and is one of the most difficult to be measured. The readiness effort to be put in studying is a product of many factors, varying from student personality and abilities to the task characteristic features, learning stimuli, the environment and the behavior of the relations (parents, teachers, classmates, etc.).[1] It is one of the most discussed phenomena in psychology and the research shows that it functions at three levels:

- At the first level, cognitive- perceptive, new impressions are collected and readiness for accepting diverse situations is shown;
- At the second level, behavioral components dominate – forming strategy for adaptation to the changing environment. The development of the intellectual abilities and personal competence are decisive in solving educational tasks.

[1] The paper is partially supported by NSF of SU "St. Kliment Ochridski"

- At the third level, the motivation for studying is expressed by the strive for better results, which enhances the personal confidence and efficiency.

All of the above lead to the conclusion that the motivation for studying is determined on the one hand by the individual characteristics and abilities for cognitive functioning, and on the other hand by the continuous dynamic interaction with the environment.

This literature review of motivation, motivation for studying in particular, shows that it is a deep personable process closely related to the identity, therefore shown through the manifestation of the identity during the educational process. Scientific research is generally devoted to analysis of the factors which facilitate the development of particular knowledge, skills and competences, rather than knowledge acquisition in general. This paper exploits the same strategy of analyzing motivation and personalization of the educational process in order to investigate their nature and interrelations.

II. RESEARCH METHODOLOGY

According to psychology, the building components of the motivational system not only supplement each other, but contradict each other – as a result of the inner personal contradiction. This phenomenon lays the foundation of the development of different theories regarding the personal motivation for learning.

According to the <u>behaviorist theory</u> of learning, the main source for motivation is the external support. Important factors to have impact on the particular behavior are interpreted via the terms award and strategies for avoiding punishment.

According to the <u>cognitive theory</u> of learning, the focus of the research on motivation is on the individual characteristics of the cognitive functioning. The important motivation factors are measured not only through the terms award and punishment [1]. The motivation processes have impact on the acquisition, transfer and use of knowledge and skills during the educational process. The relevant, timely and adequate feedback is of crucial importance for the quality and efficiency of the learning. It facilitates the student to evaluate the extent to which the goals are achieved, and to mark their success. This leads to higher personal contentment and competence and stimulates the individual development.

This paper explores the cognitive theory and its approach to learning. While the behaviorist theory researches on the observable behavior, the cognitive theory aims at the non-observable psychological processes which take place during the educational process. This choice poses the question for a related research approach.

The main methods used in pedagogy are experiments, correlations, and descriptions [1].

- The experiments set special conditions whose impact is measured.
- The correlations research on the factors in the way they exist without setting special conditions. However, they do not show the relations between these factors.
- Developmental psychology more often than not uses the observation method of research in order to identify the characteristic features of children at different ages.

  Pedagogical psychology is interested in case study as well, as it bears a unique idea or problem.

As this paper aims at research on personal motivation, the methods used are description and case study.

III. EXPERIMENT

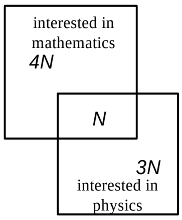
The observations and the case studies derived from them, as described below, are taken from the good practices in Bulgarian schools – mathematics studied additionally as an elective subject, schools of mathematics, and competitions in mathematics. These are widely accepted forms of education and not elite, and they gather a large number of students with interest in mathematics and motivation for learning. These classes make studying meaningful for them, enhancing the attractiveness of the mathematical task, providing optimal freedom of the students to make choices when solving the mathematical problem, thus taking the responsibility for their own development.

Example 1.

Form: School of mathematics for 4th grade.

Task: In a class, every fifth student of those interested in mathematics is also interested in physics, and every forth of those interested in physics is interested in mathematics. Only Petko and Vasko are interested in neither physics, nor mathematics. What is the total number of the students in this class if it is known that it is greater than 20 and less than 30?

Expectations: The task is set immediately after a class on the principle for switching on and off and the use of Euler-Venn's diagrams. In addition, students actively use letters as symbols in different situations in order to describe them in a mathematical manner. What is expected is a solution on the basis of the figure to the right.



The observed solution: as every fifth of these interested in mathematics is also interested in physics, their number is among the following numbers: 5, 10, 15 or 25 (as the students are less than 30). We fill in the table as follows:

No of students interested in mathematics	No of students interested in mathematics and physics	No of students interested in physics	No of students interested in mathematics or physics	No of students in the class
5	1	4	5+4–1=8	8+2=10
10	2	8	10+8–2=16	16+2=18
15	3	12	15+12–3=24	24+2=26
20	4	16	20+16–4=32	32+2=34

Therefore, the students are 26.

The student adapts the approach for solving Diophantine equations and its table representation and applies it to find a solution to this task, which is, by nature, a different one.

Example 2.

Teaching situation: The class is in an additional, non-compulsory elective course in mathematics for the 5th grade [2].

Task: What is the largest numbers of cross points of n straight lines?

Expectations: The task requires the application of the formula $\left(n.\left(n-1\right)\right):2$ concerning the number of unique combinations in twos which n items can have. The task is to internalize the requirement “the largest number of cross points”, rather than to calculate the actual number.

Solution observed: The student takes two straight lines which cross in one point. In order for the cross points to be the largest possible number, the third straight line must cross the first two lines, which makes 1 + 2 cross points. The forth straight line will respectively add 3 more cross points by crossing the first three lines, thus the number of cross points is 1 + 2 + 3. Going on to the n-th straight line will make the largest number of cross points like that: $1 + 2 + 3 + \ldots + (n - 1) = \left(n.\left(n-1\right)\right):2$.

Student prefer, and it is more natural for them, to trace the process of cross point occurrence and to induce the solution rather than to formally apply the formula.

Example 3.

Teaching situation: an autumn competition in mathematics, the task is for the 8th grade [3].

Task: Solve the equation $\lvert x-m\rvert+\lvert x+m\rvert = x$ in dependence of the parameter *m*.

Expectations: The traditional approach is opening the absolute value and making the equation a linear one. Although this approach is "scientific like", it is time consuming, requires technical skills and has a specific logical structure. Students who overcome the technical difficulties usually forget to check whether the result belongs to the set of solutions for the given equation. Another universal approach is applying the equation in the triangle or setting the two sides of the equation on the power of 2 as they can assume only non-negative values.

Solutions observed:

1. From the type of equation it follows that $x \geq 0$.

– If $m \geq 0$, then $x+m \geq 0$. The equation is $\lvert x-m\rvert + x + m = x, \lvert x-m\rvert = -m$, so $m \leq 0$. That means that the only possible value for *m* is 0, then $x = 0$.

– If $m < 0$, then $x-m > 0$, the equation is $x-m+\lvert x+m\rvert = x, \lvert x+m\rvert = m, m \geq 0$, so there is no such *m*.

2. From the type of equation it follows that $x \geq 0$. We write the equation as $\lvert x-m\rvert = x-\lvert x+m\rvert$, then $x-\lvert x+m\rvert \geq 0$, i.e. $m \leq 0$. Similarly, from $\lvert x+m\rvert = x-\lvert x-m\rvert$ it follows that $m \geq 0$. It is clear that $m = 0$ and $x = 0$.

3. Because of the symmetry of the left side with respect to *m*, it can be written as $x+\lvert m\rvert+\lvert x-m\rvert = x, \lvert m\rvert+\lvert x-m\rvert = 0$, which leads immediately to $m = 0$ and $x = 0$.

4. From $x = \lvert x-m\rvert+\lvert x+m\rvert \geq \lvert x+m\rvert$ with $m > 0$ it follows that $x+m < x$, which is impossible. From $x = \lvert x-m\rvert+\lvert x+m\rvert \geq \lvert x-m\rvert$ with $m < 0$ it follows that $x-m < x$, which is equally impossible. Therefore, the only solution is $m = 0$, which leads to $x = 0$.

These solutions show the ability of the students to skillfully interpret the task, so as to find the solution directly and in the most efficient way.

IV. CONCLUSIONS

1. At a given stage of compiling a number of rules, schemata and algorithms, there is an opportunity for a logical reorganization of the mathematical knowledge. A catalyst of this process is the motivation, which is a result of the personal freedom for individual contribution in a wide age range.
2. Students, who are motivated to study and achieve, employ a high-level cognitive processes while they learn, absorb and store more information. An important task of the teachers is to plan the means to support student motivation.
3. In order to motivate the learning and increase the satisfaction with success (as an attribute of motivation), more appropriate are moderately difficult to difficult (but not impossible) tasks rather than easy ones.
4. The general study of mathematics in our educational system is almost completely devoted to the learning (by heart) of given rules, schemata, algorithms and their application to pre-selected easy tasks. The question is whether it is possible at all, and how, to employ this "tradition" to the modern teaching methodology of learning, such as student-oriented, problem-solving, and project-oriented approaches. These methodologies are viewed as tools for motivating students for expressing themselves, as they apparently have the potential for doing so.

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ИНДИВИДУАЛНИ СПОСОБНОСТИ НА МОТИВИРАНИТЕ УЧЕНИЦИ В ПРОЦЕСА НА РЕШАВАНЕ НА МАТЕМАТИЧЕСКИ ЗАДАЧИ

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РЕЗЮМЕ

В статията се изследват индивидуалните способности на мотивираните ученици в процеса на решаване на математически задачи. Като теоретична основа е приета когнитивната теория за познавателните процеси. Използвани са описателният метод и изследване на отделен случай. Описани са някои наблюдения и са направени съответните заключения.