

MOTZKIN DECOMPOSABLE SETS

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ABSTRACT

Theodore Motzkin proved, in 1936, that any polyhedral convex set can be expressed as the (Minkowski) sum of a polytope and a polyhedral convex cone. We have provided several characterizations of the larger class of closed convex sets, Motzkin decomposable, in finite dimensional Euclidean spaces which are the sum of a compact convex set with a closed convex cone. These characterizations involve different types of representations of closed convex sets as the support functions, dual cones and linear systems whose relationships are also analyzed. The obtaining of information about a given closed convex set F and the parametric linear optimization problem with feasible set F from each of its different representations, including the Motzkin decomposition, is also discussed. Another result establishes that a closed convex set is Motzkin decomposable if and only if the set of extreme points of its intersection with the linear subspace orthogonal to its lineality is bounded. We characterize the class of the extended functions whose epigraphs are Motzkin decomposable sets showing, in particular, that these functions attain their global minima when they are bounded from below. Calculus of Motzkin decomposable sets and functions is provided.

Keywords: Motzkin decomposition, closed convex sets, convex functions
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References

- [1] Goberna M.A., E. Gonzalez, J. E., Martínez-Legaz and Todorov M.I. Motzkin decomposition of closed convex sets. *J. Math. Anal. Appl.* 364 (2010) 209-221.
- [2] Goberna M.A., J. E. Martínez-Legaz and Todorov M. I. On Motzkin decomposable sets and functions. *J. Math. Anal. Appl.* (2010), doi: 10.1016/j.jmaa.2010.0707.

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