ПЛОВДИВСКИ УНИВЕРСИТЕТ "ПАИСИЙ ХИЛЕНДАРСКИ", БЪЛГАРИЯ
НАУЧНИ ТРУДОВЕ, ТОМ 37, КН. 3, 2010 – МАТЕМАТИКА
PLOVDIV UNIVERSITY "PAISSII HILENDARSKI", BULGARIA
SCIENTIFIC WORKS, VOL. 37, BOOK 3, 2010 – MATHEMATICS

A SOR–NEKRASSOV–MEHMKE PROCEDURE FOR NUMERICAL SOLUTION OF LINEAR SYSTEMS OF EQUATIONS[1]

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Abstract. A SOR (*successive overrelaxation*) iteration procedure for finding a solution of a linear system of algebraic equations $Ax-b=0$ is given and interesting numerical examples are presented.

Key words: solving linear system of equations, Jacobi method, Richardson method, Nekrassov–Mehmke methods, SOR–Nekrassov–Mehmke method, successive overrelaxation procedure, accelerations factors
2010 Mathematics Subject Classification: 65F10

1. Introduction

Let us consider the linear system $Ax-b=0$, or

$$\begin{aligned} &a_{11}x_1+a_{12}x_2+\cdots+a_{1n}x_n-b_1=0=f_1(x_1,x_2,\dots,x_n),\\ &a_{21}x_1+a_{22}x_2+\cdots+a_{2n}x_n-b_2=0=f_2(x_1,x_2,\dots,x_n),\\ &\dots\\ &a_{n1}x_1+a_{n2}x_2+\cdots+a_{nn}x_n-b_n=0=f_n(x_1,x_2,\dots,x_n). \end{aligned} \tag{1}$$

Suppose that the matrix A is diagonally dominant and $a_{ii}>0, i=1,\dots,n$.

[1] This research has been partially supported by the project of Bulgarian National Scientific Found from 2010.

In principle any iterative method for solving system (1) can be written in matrix form as

$$X^{k+1} = DX^k + d.$$

In this paper we propose new iterative algorithms based on the classical methods of Nekrassov.

A modification of Euler - Richardson method

Using Jacobi iteration scheme (see, Björck [3]), the sequence of consecutive approximations x_i^k, is computed in this way:

$$\begin{aligned} x_i^{k+1} &= -\sum_{j\neq i}^{n} \frac{a_{ij}}{a_{ii}} x_j^k + \frac{b_i}{a_{ii}} \\ &= x_i^k - \frac{1}{a_{ii}} f_i(x_1^k, \ldots, x_n^k) \qquad (2) \\ &= x_i^k - \frac{f_i(x_1^k, \ldots, x_n^k)}{\partial f_i / \partial x_i^k}, \end{aligned}$$

$$i = 1, 2, \ldots, n; \;\; k = 0, 1, \ldots,$$

i.e. (2) is Newton scheme, applied for the equation $f_i = 0$.

The Jacobi algorithm have perfect computational properties and this method inspired a number of other contributions (see, for instance, Saad and Van der Vorst [16], Freund, Golub, Nachtigal [7], Faddeev D. and Faddeeva V. [5], Ishihara, Muroya, Yamamoto [9] and Maleev [12]).

A more powerful class of methods can be described by the recursion

$$x^{k+1} = x^k - \alpha_k \left(Ax^k - b \right), \qquad (3)$$

where $\alpha_i, \; i = 0, 1, \ldots, k$ are parameters.

The rate of convergence of Richardson method (3) depends on the spectrum of matrix A.

The determination of Richardson scaling factors using Chebyshev polynomials, or extremal polynomials can be found in Zawilski [20], Stork [19], Fischer and Reichel [6], De Boor and Rice [4].

For instance, the Richardson iteration (3) with the application of Chebyshev acceleration factors is defined by

(4) $$\alpha_i = 2\left(a + b - (b-a)\cos\frac{(2i+1)\pi}{2(k+1)}\right)^{-1},$$

$$i = 0, 1, 2, \ldots, k$$

and

$$a \le \lambda_i \le b, \ \ i = 1, \ldots, n$$

where λ_i - are the eigenvalues of matrix A.

In practice, the number of iteration steps k for receiving the solution of system (1) with fixed accuracy ϵ is not known and success of the procedure (3) depends on the proper ordering of the acceleration parameters.

The analytic analysis introduces than variance reduction for comparing of two techniques (2) and (3).

An alternative perspective is the construction of Richardson iteration, strongly depending of the data components x_i^k, $i = 1, \ldots, n$; $k = 0, 1, \ldots$.

A modification of Richardson method (assume that $x_i \neq x_j$ and $x_i^0 \neq x_j^0$ for $i \neq j$) for finding a solution of linear system of algebraic equations is given by Kyurkchiev, Petkov and Iliev [10]:

(5) $$x_i^{k+1} = x_i^k - \frac{1}{M_i^k}\left(\sum_{j=1}^{n} a_{ij}x_j^k - b_i\right),$$

$$i = 1, 2, \ldots, n; \quad k = 0, 1, 2, \ldots,$$

where

$$M_i^k = \prod_{j \neq i}^{n} |x_i^k - x_j^k|, \ \ i = 1, 2, \ldots, n; \ \ k = 0, 1, \ldots.$$

Let

$$\omega_i^k = \frac{a_{ii}}{M_i^k}, \ i = 1, 2, \ldots, n; \ k = 0, 1, \ldots.$$

Remark. One geometric interpretation of method (5) is the following.

Let x_i^k, $i = 1, \ldots, n$ are different approximations of x_i,$i = 1, \ldots, n$ and let us denote by $F_k(x)$ the polynomial

$$F_k(x) = (x - x_1^k)(x - x_2^k)\ldots(x - x_n^k).$$

Then for $x = x_i^k$, we have

$$F_k'(x_i^k) = \prod_{j \neq i}^{n} (x_i^k - x_j^k)$$

and previous expression can be used for approximation of a_{ii} (in the Jacobi method $a_{ii} = \partial f_i / \partial x_i^k$).

The following theorem is valid:

Theorem A. (Kyurkchiev, Petkov and Iliev[10]). Let

$$\omega_i^k \in (1,2), \;\; \mu_i = \sum_{j \neq i}^{n} \frac{|a_{ij}|}{a_{ii}} \in \left(0, \frac{1 - |1 - \omega_i^k|}{\omega_i^k}\right) \subset (0,1),$$

$$K_{\omega_i^k} = |1 - \omega_i^k| + \mu_i \omega_i^k \leq q < 1, \tag{6}$$

$$i = 1, 2, \ldots, n; \; k = 0, 1, 2, \ldots .$$

Then the iteration procedure (5) converges to the unique solution x_i, $i = 1, 2, \ldots, n$ of the system (1).

A modification of Nekrassov–Mehmke method

In a similar manner other iterations can be obtained which are modifications of algorithms which have been explored in details in book by Barrett, R., M. Berry and others [2].

As an example a scheme of the Gauss–Seidel or the Nekrassov method (see Nekrassov [15], Mehmke [13] and Nekrassov and Mehmke [14]) look thus:

$$x_i^{k+1} = -\sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} x_j^{k+1} - \sum_{j=i+1}^{n} \frac{a_{ij}}{a_{ii}} x_j^k + \frac{b_i}{a_{ii}}, \tag{7}$$

$$i = 1, 2, \ldots, n; \;\; k = 0, 1, 2, \ldots .$$

Here after, we shall call above scheme the *Nekrassov–Mehmke 1–method (NM1).*

A modification of Nekrassov method (assume that $x_i \neq x_j$ and $x_i^0 \neq x_j^0$ for $i \neq j$) for finding a solution of linear system of algebraic equations is given by Iliev, Kyurkchiev and Petkov [8]:

$$x_i^{k+1} = x_i^k - \frac{1}{N_i^k}\left(\sum_{j=1}^{i-1} a_{ij}x_j^{k+1} + a_{ii}x_i^k + \sum_{j=i+1}^{n} a_{ij}x_j^k - b_i\right), \tag{8}$$

$$i = 1, 2, \dots, n; \quad k = 0, 1, 2, \dots,$$

where

$$N_i^k = \prod_{j=1}^{i-1} |x_i^k - x_j^{k+1}| \prod_{j=i+1}^{n} |x_i^k - x_j^k|, \ \ i = 1, 2, \dots, n; \ \ k = 0, 1, \dots.$$

Let

$$\delta_i^k = \frac{a_{ii}}{N_i^k}, \ i = 1, 2, \dots, n; \ k = 0, 1, 2, \dots.$$

When $\delta_i^k = 1$ from (8) we obtain the Nekrassov method.

The following theorem is valid

Theorem B. (Iliev, Kyurkchiev and Petkov[8]). Let

$$\beta_i = \sum_{j=1}^{i-1} \frac{|a_{ij}|}{a_{ii}}, \ \ \gamma_i = \sum_{j=i+1}^{n} \frac{|a_{ij}|}{a_{ii}}, \ \ \delta_i^k \in (1, 2),$$

$$\beta_i + \gamma_i \in \left(0, \frac{1 - |1 - \delta_i^k|}{\delta_i^k}\right) \subset (0, 1), \tag{9}$$

$$i = 1, 2, \dots, n; \ k = 0, 1, 2, \dots.$$

Then the iteration procedure (8) converges to the unique solution x_i, $i = 1, 2, \dots, n$ of the system (1).

2. Main results

Wide area of problems and practical tasks in tomography, and image processing problems are reduced to the problem of solving a system of algebraic equations with some constraint conditions for the initial approximations

x_i^0, $i = 1, \dots, n$ (see, Björck [3], A. van der Sluis and H. van der Vorst [18], A. Louis and F. Natterer [11] and R. Santos and A. de Pierro [17]).

In a number of cases the success of the procedures of type (7) depends on the proper ordering of the equations (and x_i, $i = 1, \dots, n$) in system (1).

In spite of this fact the following modification of the Nekrassov method is known (see Faddeev, D. and Faddeeva, V. [5]):

$$x_i^{k+1} = -\sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} x_j^k - \sum_{j=i+1}^{n} \frac{a_{ij}}{a_{ii}} x_j^{k+1} + \frac{b_i}{a_{ii}}, \tag{10}$$

$$i = 1, 2, \dots, n; \;\; k = 0, 1, 2, \dots.$$

Here after, we shall call above scheme the *Nekrassov–Mehmke 2–method (NM2)*.

Let us explore the following modification of the method (10) (assume that $x_i \neq x_j$ and $x_i^0 \neq x_j^0$ for $i \neq j$):

$$x_i^{k+1} = x_i^k - \frac{1}{D_i^k} \left(\sum_{j=1}^{i-1} a_{ij} x_j^k + a_{ii} x_i^k + \sum_{j=i+1}^{n} a_{ij} x_j^{k+1} - b_i \right), \tag{11}$$

$$i = 1, 2, \dots, n; \quad k = 0, 1, 2, \dots,$$

where

$$D_i^k = \prod_{j=1}^{i-1} |x_i^k - x_j^k| \prod_{j=i+1}^{n} |x_i^k - x_j^{k+1}|, \;\; i = 1, 2, \dots, n; \;\; k = 0, 1, \dots.$$

Let

$$\lambda_i^k = \frac{a_{ii}}{D_i^k}, \; i = 1, 2, \dots, n; \; k = 0, 1, 2, \dots.$$

The SOR (*successive overrelaxation*) iteration procedure (11) can be rewritten as:

$$\begin{aligned} x_i^{k+1} &= x_i^k - \frac{a_{ii}}{D_i^k} \left(\sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} x_j^k + x_i^k + \sum_{j=i+1}^{n} \frac{a_{ij}}{a_{ii}} x_j^{k+1} - \frac{b_i}{a_{ii}} \right) \\ &= x_i^k (1 - \lambda_i^k) - \lambda_i^k \left(\sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} x_j^k + \sum_{j=i+1}^{n} \frac{a_{ij}}{a_{ii}} x_j^{k+1} - \frac{b_i}{a_{ii}} \right). \end{aligned} \tag{12}$$

When $\lambda_i^k = 1$ from (11) we obtain the method (10).
We give a convergence theorem for the relaxation method (11).

Theorem 1. Let

(13)
$$\beta_i = \sum_{j=1}^{i-1} \frac{|a_{ij}|}{a_{ii}},\ \ \gamma_i = \sum_{j=i+1}^{n} \frac{|a_{ij}|}{a_{ii}},\ \ \lambda_i^k \in (1,2),$$
$$\beta_i + \gamma_i \in \left(0, \frac{1-|1-\lambda_i^k|}{\lambda_i^k}\right) \subset (0,1),$$
$$i = 1,2,\dots,n;\ k = 0,1,2,\dots.$$

Then the iteration procedure (11) converges to the unique solution x_i, $i = 1,2,\dots,n$ of the system (1).

Proof. Following the ideas given in paper by Iliev, Kyurkchiev and Petkov [8] for the error $x_i^{k+1} - x_i$, we have

(14)
$$\begin{aligned} x_i^{k+1} - x_i &= x_i^k(1-\lambda_i^k) - x_i - \lambda_i^k \left(\sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} x_j^k + \sum_{j=i+1}^{n} \frac{a_{ij}}{a_{ii}} x_j^{k+1} - \right. \\ &\left. - \sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} x_j - \sum_{j=i+1}^{n} \frac{a_{ij}}{a_{ii}} x_j - x_i \right) \\ &= (x_i - x_i^k)(\lambda_i^k - 1) + \lambda_i^k \sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} (x_j - x_j^k) + \lambda_i^k \sum_{j=i+1}^{n} \frac{a_{ij}}{a_{ii}} (x_j - x_j^{k+1}) \end{aligned}$$

and

(15)
$$\begin{aligned} |x_i^{k+1} - x_i| &\leq |\lambda_i^k - 1||x_i^k - x_i| + \lambda_i^k \sum_{j=1}^{i-1} \frac{|a_{ij}|}{a_{ii}} |x_j - x_j^k| + \lambda_i^k \sum_{j=i+1}^{n} \frac{|a_{ij}|}{a_{ii}} |x_j - x_j^{k+1}| \\ &\leq |\lambda_i^k - 1|||x - x^k||_1 + \lambda_i^k \beta_i ||x - x^k||_1 + \lambda_i^k \gamma_i ||x - x^{k+1}||_1 \\ &= \left(|\lambda_i^k - 1| + \beta_i \lambda_i^k\right) ||x - x^k||_1 + \lambda_i^k \gamma_i ||x - x^{k+1}||_1. \end{aligned}$$

Let

$$\max_i |x_i^{k+1} - x_i| = |x_{i_0}^{k+1} - x_{i_0}|.$$

Then from (15) we get

$$\begin{aligned} ||x - x^{k+1}||_1 &= \max_i |x_i - x_i^{k+1}| = |x_{i_0}^{k+1} - x_{i_0}| \\ &\leq \left(|\lambda_{i_0}^k - 1| + \beta_{i_0}\lambda_{i_0}^k\right) ||x - x^k||_1 + \lambda_{i_0}^k \gamma_{i_0} ||x - x^{k+1}||_1 \end{aligned}$$

and

$$(16) \qquad ||x - x^{k+1}||_1 \leq \frac{|\lambda_{i_0}^k - 1| + \beta_{i_0}\lambda_{i_0}^k}{1 - \lambda_{i_0}^k \gamma_{i_0}} ||x - x^k||_1 = K_{i_0}^* ||x - x^k||_1.$$

Evidently from (13) we have

$$K_{i_0}^* = \frac{|\lambda_{i_0}^k - 1| + \beta_{i_0}\lambda_{i_0}^k}{1 - \lambda_{i_0}^k \gamma_{i_0}} \leq \frac{|\lambda_{i_0}^k - 1| + \lambda_{i_0}^k \left(\frac{1 - |\lambda_{i_0}^k - 1|}{\lambda_{i_0}^k} - \gamma_{i_0}\right)}{1 - \lambda_{i_0}^k \gamma_{i_0}} = 1.$$

This proves Theorem 1.

3. Numerical example 1

As an example we will consider the system:

$$\left| \begin{aligned} x_1 + 3x_2 - 2x_3 &= 5 \\ 3x_1 + 5x_2 + 6x_3 &= 7 \\ 2x_1 + 4x_2 + 3x_3 &= 8 \end{aligned} \right.$$

The exact solution of the system is $x(-15, 8, 2)$.

For an initial approximation we choose $x^0(-15.02, 8.02, 2.02)$.

We give the results of numerical experiments (8 iterations) for each of methods (10) and (11).

Table 1

0	$X[3] = 2.02000000000000$ $X[2] = 8.02000000000000$ $X[1] = -15.02000000000000$	$Y[3] = 2.02000000000000$ $Y[2] = 8.02000000000000$ $Y[1] = -15.02000000000000$
1	$X[3] = 2.01902190923318$ $X[2] = 8.01888522617379$ $X[1] = -15.01999646387891$	$Y[3] = 1.98666666666667$ $Y[2] = 8.02800000000000$ $Y[1] = -15.11066666666667$
2	$X[3] = 2.01811599031574$ $X[2] = 8.01784991599005$ $X[1] = -15.01998963954672$	$Y[3] = 2.03644444444444$ $Y[2] = 8.02266666666667$ $Y[1] = -14.99511111111112$
3	$X[3] = 2.01727696746792$ $X[2] = 8.01688823891855$ $X[1] = -15.01997975673980$	$Y[3] = 1.96651851851852$ $Y[2] = 8.03724444444445$ $Y[1] = -15.17869629629630$
4	$X[3] = 2.01649994986106$ $X[2] = 8.01599478968974$ $X[1] = -15.01996702833352$	$Y[3] = 2.06947160493827$ $Y[2] = 8.02385185185186$ $Y[1] = -14.93261234567904$
5	$X[3] = 2.01578040372231$ $X[2] = 8.01516455747490$ $X[1] = -15.01995165156159$	$Y[3] = 1.92327242798355$ $Y[2] = 8.05164049382717$ $Y[1] = -15.30837662551440$
6	$X[3] = 2.01511412643429$ $X[2] = 8.01439289727067$ $X[1] = -15.01993380914837$	$Y[3] = 2.13673042524005$ $Y[2] = 8.02094946502058$ $Y[1] = -14.78938754458166$
7	$X[3] = 2.01449722249096$ $X[2] = 8.01367550333466$ $X[1] = -15.01991367036020$	$Y[3] = 1.83165907636033$ $Y[2] = 8.07564163511660$ $Y[1] = -15.56360675262915$
8	$X[3] = 2.01392608117971$ $X[2] = 8.01300838452836$ $X[1] = -15.01989139198147$	$Y[3] = 2.27488232159730$ $Y[2] = 8.00830526566073$ $Y[1] = -14.47515115378761$

In Table 1 the following notations are used:

– in the first column the serial number of the iteration is given;

– using the modified Nekrassov–Mehmke scheme (11) in the second column the obtained results are given (array $x[\,]$);

– using the Nekrassov–Mehmke scheme (NM2) - (10) in the third column the obtained results are given (array $y[\,]$).

4. Remarks

1. One geometric interpretation of method (11) is the following.

Let us denote by $F_k(x)$ the polynomial

$$F_k(x) = (x - x_1^k)(x - x_2^k)\dots(x - x_i^k)(x - x_{i+1}^{k+1})(x - x_{i+2}^{k+1})\dots(x - x_n^{k+1}).$$

Then for $x = x_i^k$, we have

$$F_k'(x_i^k) = \prod_{j=1}^{i-1}(x_i^k - x_j^k)\prod_{j=i+1}^{n}(x_i^k - x_j^{k+1}).$$

In spite of this fact the previous expression can be used for approximation of a_{ii} in the SOR–Nekrassov procedure.

2. Let in the algorithm (11) we choose

$$D_i^k = \max\left\{a_{ii},\ \prod_{j=1}^{i-1}(x_i^k - x_j^k)\prod_{j=i+1}^{n}(x_i^k - x_j^{k+1})\right\},$$
$$i = 1, 2, \dots, n;\ k = 0, 1, 2, \dots,$$

where the sign of product is defined to be equal to the sign of a_{ii}.

This leads to the *improved SOR–Nekrassov–Mehmke method*.

Numerical example 2

As an example we will consider the system:

$$\left|\begin{array}{l} x_1 - 0.1x_2 = 0.8 \\ 7x_1 + x_2 = 9. \end{array}\right.$$

The exact solution of system is $x(1, 2)$.

For initial approximation we choose $x^0(0.9, 1.8)$.

Table 2

1	$XNM2[1] = 0.994444$ $X[2] = 2.7$	$NM2[1] = 1.07$ $NM2[2] = 2.7$
2	$XNM2[1] = 1.02312$ $XNM2[2] = 2.31238$	$NM2[1] = 0.951$ $NM2[2] = 1.51$
3	$X[1] = 0.994456$ $XNM2[2] = 1.94456$	$NM2[1] = 1.0343$ $NM2[2] = 2.343$
4	$XNM2[1] = 1.00348$ $X[2] = 2.03881$	$NM2[1] = 0.97599$ $NM2[2] = 1.7599$
5		$NM2[1] = 1.01681$ $NM2[2] = 2.16807$
6		$NM2[1] = 0.988233$ $NM2[2] = 1.88233$
7		$NM2[1] = 1.00824$ $NM2[2] = 2.08237$
8		$NM2[1] = 0.994232$ $NM2[2] = 1.94232$
9		$NM2[1] = 1.00404$ $NM2[2] = 2.04038$

In Table 2 we give the results of numerical experiments.
The following notations are used:

– in the first column a serial number of iteration step is used;

– in the third column, with array $NM2[]$, $i = 1, 2$ received results are denoted, using classical (NM2) scheme (10);

– in second columns results are given, using modified scheme (11).

Two arrays are necessary $X[]$ and $XNM2[]$ as follows:

in $X[]$– data are stored, when is fulfilled $D_i^k = a_{ii}$, and

in $XNM2[]$ – data are stored, when $D_i^k = \prod_{j=1}^{i-1}(x_i^k - x_j^k) \prod_{j=i+1}^{n} (x_i^k - x_j^{k+1})$, where the sign of product is defined to be equal to the sign of a_{ii}.

From given results it can be seen that components x_1 and x_2 are calculated by using both schemes.

It turns out that using NM2 method (10) 9 iteration steps for receiving the solution with fixed accuracy ϵ are necessary.

For the same precision the modified scheme given here consummates only 4 iterations.

3. Numerical experiments demonstrate that in some aspects improved convergence can be reached through mentioned above combined "NM2–improved SOR–Nekrassov–Mehmke" iteration procedure.

4. In [5] D. Faddeev and V. Faddeeva especially pointed out that of certain interest are such iteration processes in which cycles studied in two Nekrassov methods (7) and (10) are alternate.

The modified Nekrassov method (10) possesses not better convergence in comparison with method (7).

But if matrix A is positive definite then eigenvalues of the matrix G in the matrix equations $x = Gx+t$ are real and this allows to apply different methodic for improving rate of convergence, i.e. as an example Abramov's technique [1].

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Received 30 September 2010

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УСКОРЕНИ МЕТОДИ ОТ ТИП НЕКРАСОВ-МЕМКЕ ЗА ЧИСЛЕНО РЕШАВАНЕ НА ЛИНЕЙНИ СИСТЕМИ УРАВНЕНИЯ

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Резюме. Изследвана е една ускорена итерационна процедура за числено решаване на линейна система от алгебрични уравнения $Ax - b = 0$ и са представени интересни числени експерименти.