ПЛОВДИВСКИ УНИВЕРСИТЕТ "ПАИСИЙ ХИЛЕНДАРСКИ", БЪЛГАРИЯ
НАУЧНИ ТРУДОВЕ, ТОМ 37, КН. 3, 2010 – МАТЕМАТИКА
PLOVDIV UNIVERSITY "PAISSII HILENDARSKI", BULGARIA
SCIENTIFIC WORKS, VOL. 37, BOOK 3, 2010 – MATHEMATICS

COMPUTING CLASSES OF ISOMORPHIC NEAR-RINGS ON CYCLIC GROUPS OF ORDER UP TO 23

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Abstract. All classes of isomorphic near-rings on $\mathbb{Z}_n$, $n \leq 23$ are computed. These near-rings are checked for the right distributive law and examined for some other properties.

Key words: near-ring, finite cyclic group
Mathematics Subject Classification 2000: 16Y30

1. Introduction

An algebraic system $(G, +, *)$ is a *(left) near-ring* on $(G, +)$ if $(G, +)$ is a group, $(G, *)$ is a semigroup and $a * (b + c) = a * b + a * c$ for $a, b, c \in G$. The left distributive law yields $x * 0 = 0$ for $x \in G$. A near-ring $(G, +, *)$ is called *zero-symmetric*, if $0 * x = 0$ holds for $x \in G$.

J. R. Clay initiated the study of near-rings whose additive groups are finite cyclic ones in 1964 [1]. Some sufficient conditions for the construction of near-rings on any finite cyclic groups were obtained.

We will assume G coincides with the set $\mathbb{Z}_n = \{0, 1, \ldots, n-1\}$, $2 \leq n < \infty$ since every cyclic group of order n is isomorphic to the group of the remainders of modulo n. We will denote the functions mapping $\mathbb{Z}_n$ into itself by π, and the addition and the multiplication modulo n we will denote by $+$ and $\cdot$ respectively. The equality $c = a \cdot b$ will be equivalent to the congruence $ab \equiv c \pmod{n}$.

It is known [1] that there exists a bijective correspondence between the left distributive binary operations $*$ defined on $\mathbb{Z}_n$ and the n^n functions π mapping $\mathbb{Z}_n$ into itself. If $r*1=b$ defines the function $\pi(r)=b$, then according to [1, Theorem II], the binary operation $*$ is left distributive exactly when, for any $x,y\in\mathbb{Z}_n$, the equality

$$\pi(x)\cdot\pi(y)=\pi(x\cdot\pi(y)) \tag{1}$$

holds.

According to the above result, obtaining the near-rings on $\mathbb{Z}_n$ is equivalent to obtaining functions π such that equation (1) holds.

Let f be a group automorphism on $(\mathbb{Z}_n,+)$, and suppose that $f(1)=s$, where $(n,s)=1$. Assume π_1 and π_2 define two associative operations $*_1$ and $*_2$ respectively. Then by [1, Theorem III] f is a near-ring isomorphism iff

$$\pi_1(p)=\pi_2(p\cdot s)\ , \tag{2}$$

for all $p\in\mathbb{Z}_n$.

We use the equation (2) to obtain the non-isomorphic (classes of isomorphic) near-rings on $\mathbb{Z}_n$.

2. Algorithm for computing non-isomorphic near-rings and data structure

2.1. Data structure. We use the following notation for the near-rings

$$k)\ (\ x_0\ x_1\ \ldots\ x_{n-1}\ )\ , \tag{3}$$

where k is the number of the generated near-ring and x_i are the values of the function π: $x_i=\pi(i)$, $i\in\mathbb{Z}_n$.

For example, "2) (0 0 0 1)" means it is the second near-ring on $\mathbb{Z}_4$ with values of the function π: $\pi(0)=\pi(1)=\pi(2)=0,\ \pi(3)=1$.

Example for operations '+' and '' into a near-ring.* The table for addition is the same as the table for addition in $\mathbb{Z}_n$. The table for multiplication in a near-ring is obtained by using the rule for multiplication in a near-ring: $p*q=\pi(p)\cdot q$.

For the near-ring

$$63)\ (\ 0\ 5\ 4\ 0\ 2\ 1\ )$$

on $\mathbb{Z}_6$, the tables for addition and multiplication are the following

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

*	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	5	4	3	2	1
2	0	4	2	0	4	2
3	0	0	0	0	0	0
4	0	2	4	0	2	4
5	0	1	2	3	4	5

For this near-ring the right distributive law fails:

$$(1+2)*1=\pi(3)\cdot 1=0\cdot 1=0\neq 3=5+4=\pi(1)\cdot 1+\pi(2)\cdot 1=1*1+2*1\,.$$

2.2. Algorithm for computing non-isomorphic near-rings

All classes of isomorphic near-rings on cyclic groups of order up to 15 were published in [2]. All classes of isomorphic near-rings (non-isomorphic near-rings) on cyclic groups of order up to 23 are computed by using the following algorithm.

Non-isomorphic near-rings and the corresponding classes can be solved in linear time regarding the number of all near-rings.

Description of the algorithm. We are working with already generated near-rings on $\mathbb{Z}_n$ for fixed $n \leq 23$. An one-dimensional array is used to save a near-ring with reference to the notation (3). The near-rings are read from a previously generated file. Procedure *"nisom_nr"* is executed for each consecutive near-ring.

The software system can compute non-isomorphic near-rings from the sets of near-rings corresponding to the constructions given in the theorems for lower bounds. The near-rings from these sets are generated consecutively and each near-ring is tested with the procedure *"nisom_nr"*. We do this because some sets of near-rings are very large and there will be a big delay, if these near-rings are read from a file.

Procedure "nisom_ nr". Input parameter: a near-ring "*nr*".

Using the table generated in advance (procedure *"compute_ isom_ table"*) with automorphisms we find all isomorphic near-rings to the near-ring *nr*.

We can obtain the same near-rings corresponding to the different automorphisms. In this case we get only one near-ring.

The found isomorphic near-rings are compared with *nr* and if one of them is less lexicographic than the *nr* (lexicographic order corresponds to the notation(3)), then the near-ring *nr* is isomorphic to the previous processed near-ring and we discard the near-ring *nr*.

In the other case, *nr* is the first lexicographic (non-isomorphic) near-ring from the class of isomorphic near-rings. For *nr* it finds the number of different isomorphic near-rings and the number of the automorphisms that have produced these isomorphic near-rings.

The result is displayed as: consequtive number, the notation (3) of the near-ring and a list of number of automorphisms that have produced the isomorphic near-rings in their class.

Procedure "compute_ isom_ table". Input parameter: "n".

Let s be a relatively prime number with n. For each s we find the corresponding automorphism $0 \cdot s, 1 \cdot s, 2 \cdot s, \ldots, (n-1) \cdot s$ defined with equation (2). The results are saved in the two-dimensional array `''isom''`, where `isom[s][n] = 1` and `isom[s][i] = i*s mod n` for $(s, n) = 1$ and $i = 0, 1, \ldots, n-1$. We use this presentation because the access to `isom[s][p]` is faster than the calculation of `p*s mod n`.

3. Computed classes of isomorphic near-rings on cyclic groups of order up to 23

	Number of near-rings	Number of classes of isomorphic near-rings
$\mathbb{Z}_3$	7	5
$\mathbb{Z}_4$	17	12
$\mathbb{Z}_5$	29	10
$\mathbb{Z}_6$	98	60
$\mathbb{Z}_7$	112	24
$\mathbb{Z}_8$	350	135
$\mathbb{Z}_9$	1170	222
$\mathbb{Z}_{10}$	1200	329
$\mathbb{Z}_{11}$	1312	139
$\mathbb{Z}_{12}$	5522	1749
$\mathbb{Z}_{13}$	5264	454
$\mathbb{Z}_{14}$	15761	2716
$\mathbb{Z}_{15}$	27998	3817
$\mathbb{Z}_{16}$	16834654	2114460
$\mathbb{Z}_{17}$	72817	4572
$\mathbb{Z}_{18}$	15642899	2610019
$\mathbb{Z}_{19}$	286381	15957
$\mathbb{Z}_{20}$	986766	128966
$\mathbb{Z}_{21}$	1468857	124447
$\mathbb{Z}_{22}$	3336633	334065
$\mathbb{Z}_{23}$	4371616	198808

Table 1. The number of non-isomorphic near-rings on $\mathbb{Z}_n$

The first part of the table gives the known number of classes of isomorphic near-rings.

The second part of the table presents the numbers of the computed classes of isomorphic near-rings on cyclic groups of order ≤ 23.

All computed classes of isomorphic near-rings on cyclic groups of order up to 23 are presented in the site of Faculty of Mathematics and Informatics, Plovdiv University, Bulgaria: `http://nearrings.fmi-plovdiv.org`

The computed classes of isomorphic near-rings are separated from the number of the isomorphic near-rings in the class.

The number of the isomorphic near-rings in the isomorphism class uniquely corresponds to the number of the divisors of $\varphi(n)$.

	Isomorph. classes	Classes of isomorphic near-rings / isomorphic near-rings in the class					
$\mathbb{Z}_3$	5	$3_{/1}$	$2_{/2}$				
$\mathbb{Z}_4$	12	$7_{/1}$	$5_{/2}$				
$\mathbb{Z}_5$	10	$3_{/1}$	$1_{/2}$	$6_{/4}$			
$\mathbb{Z}_6$	60	$22_{/1}$	$38_{/2}$				
$\mathbb{Z}_7$	24	$3_{/1}$	$2_{/2}$	$3_{/3}$	$16_{/6}$		
$\mathbb{Z}_8$	135	$18_{/1}$	$68_{/2}$	$49_{/4}$			
$\mathbb{Z}_9$	222	$9_{/1}$	$18_{/2}$	$15_{/3}$	$180_{/6}$		
$\mathbb{Z}_{10}$	329	$22_{/1}$	$25_{/2}$	$282_{/41}$			
$\mathbb{Z}_{11}$	139	$3_{/1}$	$2_{/2}$	$7_{/5}$	$127_{/10}$		
$\mathbb{Z}_{12}$	1749	$106_{/1}$	$578_{/2}$	$1065_{/4}$			
$\mathbb{Z}_{13}$	454	$3_{/1}$	$1_{/2}$	$3_{/3}$	$6_{/4}$	$11_{/6}$	$430_{/12}$
$\mathbb{Z}_{14}$	2716	$22_{/1}$	$38_{/2}$	$91_{/3}$	$2565_{/6}$		
$\mathbb{Z}_{15}$	3817	$22_{/1}$	$82_{/2}$	$473_{/4}$	$3240_{/8}$		
$\mathbb{Z}_{16}$	2114460	$58_{/1}$	$626_{/2}$	$19216_{/4}$	$2094560_{/8}$		
$\mathbb{Z}_{17}$	4572	$3_{/1}$	$1_{/2}$	$3_{/4}$	$30_{/8}$	$4535_{/16}$	
$\mathbb{Z}_{18}$	2610019	$194_{/1}$	$1815_{/2}$	$2995_{/3}$	$2605015_{/6}$		
$\mathbb{Z}_{19}$	15957	$3_{/1}$	$2_{/2}$	$2_{/3}$	$13_{/6}$	$64_{/9}$	$15873_{/18}$
$\mathbb{Z}_{20}$	128966	$106_{/1}$	$502_{/2}$	$10302_{/4}$	$118056_{/8}$		
$\mathbb{Z}_{21}$	124447	$22_{/1}$	$102_{/2}$	$91_{/3}$	$125_{/4}$	$3571_{/6}$	$120536_{/12}$
$\mathbb{Z}_{22}$	334065	$22_{/1}$	$38_{/2}$	$703_{/5}$	$333302_{/10}$		
$\mathbb{Z}_{23}$	198808	$3_{/1}$	$2_{/2}$	$187_{/11}$	$198616_{/22}$		

Table 2. The number of classes of isomorphic near-rings over the number of isomorphic near-rings in the class

Example. All near-rings on $\mathbb{Z}_6$ presented in the notation (3):

1	(0 0 0 0 0 0)
2	(0 0 0 0 0 1)
3	(0 0 0 0 1 0)
4	(0 0 0 0 1 1)
5	(0 0 0 1 0 0)
6	(0 0 0 1 0 1)
7	(0 0 0 1 1 0)
8	(0 0 0 1 1 1)
9	(0 0 1 0 0 0)
10	(0 0 1 0 0 1)
11	(0 0 1 0 1 0)
12	(0 0 1 0 1 1)
13	(0 0 1 0 5 0)
14	(0 0 1 1 0 0)
15	(0 0 1 1 0 1)
16	(0 0 1 1 1 0)
17	(0 0 1 1 1 1)
18	(0 0 4 0 0 1)
19	(0 0 4 0 0 4)
20	(0 0 5 0 1 0)
21	(0 1 0 0 0 0)
22	(0 1 0 0 0 1)
23	(0 1 0 0 0 5)
24	(0 1 0 0 1 0)
25	(0 1 0 0 1 1)
26	(0 1 0 0 4 0)
27	(0 1 0 1 0 0)
28	(0 1 0 1 0 1)
29	(0 1 0 1 1 0)
30	(0 1 0 1 1 1)
31	(0 1 0 3 0 1)
32	(0 1 0 3 0 3)
33	(0 1 0 3 0 5)
34	(0 1 0 3 4 3)
35	(0 1 1 0 0 0)
36	(0 1 1 0 0 1)
37	(0 1 1 0 1 0)
38	(0 1 1 0 1 1)
39	(0 1 1 0 5 5)
40	(0 1 1 1 0 0)
41	(0 1 1 1 0 1)
42	(0 1 1 1 1 0)
43	(0 1 1 1 1 1)
44	(0 1 2 0 4 2)
45	(0 1 2 0 4 5)
46	(0 1 2 3 4 5)
47	(0 1 4 0 4 1)
48	(0 1 4 0 4 4)
49	(0 1 4 3 4 1)
50	(0 1 5 0 1 5)
51	(0 2 4 0 2 1)
52	(0 2 4 0 2 4)
53	(0 3 0 3 0 1)
54	(0 3 0 3 0 3)
55	(0 3 4 3 0 1)
56	(0 4 0 0 4 0)
57	(0 4 2 0 4 2)
58	(0 4 4 0 4 1)
59	(0 4 4 0 4 4)
60	(0 5 0 0 0 1)
61	(0 5 0 3 0 1)
62	(0 5 1 0 5 1)
63	(0 5 4 0 2 1)
64	(0 5 4 3 2 1)
65	(0 5 5 0 1 1)
66	(1 1 1 1 1 1)
67	(3 1 1 3 1 1)
68	(3 1 1 3 1 3)
69	(3 1 1 3 3 1)
70	(3 1 1 3 3 3)
71	(3 1 1 3 5 5)
72	(3 1 3 3 1 1)
73	(3 1 3 3 1 3)
74	(3 1 3 3 3 1)
75	(3 1 3 3 3 3)
76	(3 1 3 3 3 5)
77	(3 1 5 3 1 5)
78	(3 3 1 3 1 1)
79	(3 3 1 3 1 3)
80	(3 3 1 3 3 1)
81	(3 3 1 3 3 3)
82	(3 3 1 3 5 3)
83	(3 3 3 3 1 1)
84	(3 3 3 3 1 3)
85	(3 3 3 3 3 1)
86	(3 3 3 3 3 3)
87	(3 3 5 3 1 3)
88	(3 5 1 3 5 1)
89	(3 5 3 3 3 1)
90	(3 5 5 3 1 1)
91	(4 1 4 1 4 1)
92	(4 1 4 1 4 4)
93	(4 1 4 4 4 1)
94	(4 1 4 4 4 4)
95	(4 4 4 1 4 1)
96	(4 4 4 1 4 4)
97	(4 4 4 4 4 1)
98	(4 4 4 4 4 4)

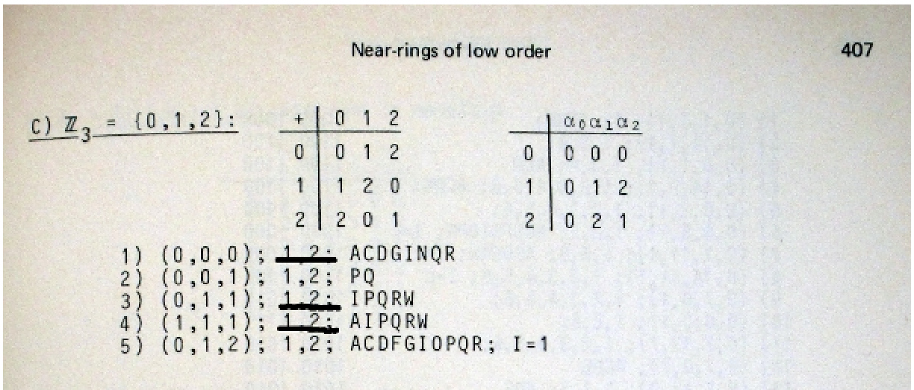
The non-isomorphic near-rings on $\mathbb{Z}_6$:

```
1) ( 0 0 0 0 0 0 ); 1
2) ( 0 0 0 0 0 1 ); 1,5
3) ( 0 0 0 0 1 0 ); 1,5
4) ( 0 0 0 0 1 1 ); 1,5
5) ( 0 0 0 1 0 0 ); 1
6) ( 0 0 0 1 0 1 ); 1,5
7) ( 0 0 0 1 1 0 ); 1,5
8) ( 0 0 0 1 1 1 ); 1,5
9) ( 0 0 1 0 0 1 ); 1,5
10) ( 0 0 1 0 1 0 ); 1
11) ( 0 0 1 0 1 1 ); 1,5
12) ( 0 0 1 0 5 0 ); 1,5
13) ( 0 0 1 1 0 1 ); 1,5
14) ( 0 0 1 1 1 0 ); 1
15) ( 0 0 1 1 1 1 ); 1,5
16) ( 0 0 4 0 0 1 ); 1,5
17) ( 0 0 4 0 0 4 ); 1,5
18) ( 0 1 0 0 0 1 ); 1
19) ( 0 1 0 0 0 5 ); 1,5
20) ( 0 1 0 0 1 1 ); 1,5
21) ( 0 1 0 1 0 1 ); 1
22) ( 0 1 0 1 1 1 ); 1,5
23) ( 0 1 0 3 0 1 ); 1
24) ( 0 1 0 3 0 3 ); 1,5
25) ( 0 1 0 3 0 5 ); 1,5
26) ( 0 1 0 3 4 3 ); 1,5
27) ( 0 1 1 0 1 1 ); 1
28) ( 0 1 1 0 5 5 ); 1,5
29) ( 0 1 1 1 1 1 ); 1
30) ( 0 1 2 0 4 2 ); 1,5
31) ( 0 1 2 0 4 5 ); 1,5
32) ( 0 1 2 3 4 5 ); 1,5
33) ( 0 1 4 0 4 1 ); 1
34) ( 0 1 4 0 4 4 ); 1,5
35) ( 0 1 4 3 4 1 ); 1
36) ( 0 1 5 0 1 5 ); 1,5
37) ( 0 2 4 0 2 4 ); 1,5
38) ( 0 3 0 3 0 3 ); 1
39) ( 0 4 4 0 4 4 ); 1
40) ( 1 1 1 1 1 1 ); 1
41) ( 3 1 1 3 1 1 ); 1
42) ( 3 1 1 3 1 3 ); 1,5
43) ( 3 1 1 3 3 1 ); 1,5
44) ( 3 1 1 3 3 3 ); 1,5
45) ( 3 1 1 3 5 5 ); 1,5
46) ( 3 1 3 3 1 3 ); 1,5
47) ( 3 1 3 3 3 1 ); 1
48) ( 3 1 3 3 3 3 ); 1,5
49) ( 3 1 3 3 3 5 ); 1,5
50) ( 3 1 5 3 1 5 ); 1,5
51) ( 3 3 1 3 1 3 ); 1
52) ( 3 3 1 3 3 3 ); 1,5
53) ( 3 3 1 3 5 3 ); 1,5
54) ( 3 3 3 3 3 3 ); 1
55) ( 4 1 4 1 4 1 ); 1
56) ( 4 1 4 1 4 4 ); 1,5
57) ( 4 1 4 4 4 1 ); 1
58) ( 4 1 4 4 4 4 ); 1,5
59) ( 4 4 4 1 4 4 ); 1
60) ( 4 4 4 4 4 4 ); 1
```

The numbers after every non-isomorphic near-ring are the numbers of the isomorphisms which generate the isomorphic near-rings into the isomorphism class.

Technical mistakes in [3, p.407] are found.
The non-isomorphic near-rings (isomorphism classes) on $\mathbb{Z}_3$ must be:

```
1) (0 , 0 , 0); 1; ...
2) (0 , 0 , 1); 1,2; ...
3) (0 , 1 , 1); 1; ...
4) (0 , 1 , 2); 1,2; ...
5) (1 , 1 , 1); 1; ...
```



Near-rings of low order 407

c) $\mathbb{Z}_3$ = {0,1,2}:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

	α_0	α_1	α_2
0	0	0	0
1	0	1	2
2	0	2	1

```
1) (0,0,0); 1,2; ACDGINQR
2) (0,0,1); 1,2; PQ
3) (0,1,1); 1,2; IPQRW
4) (1,1,1); 1,2; AIPQRW
5) (0,1,2); 1,2; ACDFGIOPQR; I=1
```

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4. Other tools for examination of near-rings

We have developed the library of functions that examine several properties of the generated near-rings or of the generated non-isomorphic near-rings.

A distributive near-ring is a near-ring for which both the left and the right distributive laws yield. We check the right distributive law for the generated near-rings

$$(a+b)*c = a*b + a*c\,,$$

for all $a, b, c \in \mathbb{Z}_n$. We give an example for operation $+$ and $*$ in Section 2.1.

We use the following notation for the distributive near-rings

$$k)\ (\ x_0\ x_1\ \ldots\ x_{n-1}\ );\ t\ ,$$

where k is the number of the class of isomorphic distributive near-rings, x_i are the values of the function $\pi\colon x_i = \pi(i)$, $i \in \mathbb{Z}_n$, and t is the number of the isomorphic near-rings in that class.

Here follows the list of the clases of isomorphic distributive near-rings on $\mathbb{Z}_n$, $n \leq 23$.

```
n = 3
1) ( 0 0 0 ); 1
2) ( 0 1 2 ); 2; I=1
  1 - automorphism 1
  2 - automorphisms 1,2

n = 4
1) ( 0 0 0 0 ); 1
2) ( 0 1 2 3 ); 2; I=1
3) ( 0 2 0 2 ); 1
  1 - automorphism 1
  2 - automorphisms 1,3

n = 5
1) ( 0 0 0 0 0 ); 1
2) ( 0 1 2 3 4 ); 4; I=1
  1 - automorphism 1
  2 - automorphisms 1,2,3,4

n = 6
1) ( 0 0 0 0 0 0 ); 1
2) ( 0 1 2 3 4 5 ); 2; I=1
3) ( 0 2 4 0 2 4 ); 2
4) ( 0 3 0 3 0 3 ); 1
  1 - automorphism 1
  2 - automorphisms 1,5

n = 7
1) ( 0 0 0 0 0 0 0 ); 1
2) ( 0 1 2 3 4 5 6 ); 6; I=1
  1 - automorphism 1
  2 - automorphisms 1,2,3,4,5

n = 8
1) ( 0 0 0 0 0 0 0 0 ); 1
2) ( 0 1 2 3 4 5 6 7 ); 4; I=1
3) ( 0 2 4 6 0 2 4 6 ); 2
4) ( 0 4 0 4 0 4 0 4 ); 1
  1 - automorphism 1
  2 - automorphisms 1,3
  4 - automorphisms 1,3,5,7

n = 9
1) ( 0 0 0 0 0 0 0 0 0 ); 1
2) ( 0 1 2 3 4 5 6 7 8 ); 6; I=1
3) ( 0 3 6 0 3 6 0 3 6 ); 2
  1 - automorphism 1
  2 - automorphisms 1,2
  6 - automorphisms 1,2,4,5,7,8

n = 10
1) (0 0 0 0 0 0 0 0 0 0); 1
2) (0 1 2 3 4 5 6 7 8 9); 4; I=1
3) (0 2 4 6 8 0 2 4 6 8); 4
4) (0 5 0 5 0 5 0 5 0 5); 1
  1 - automorphism 1
  4 - automorphisms 1,3,7,9
```

```
n = 11
1) ( 0 0 0 0 0 0 0 0 0 0  0 ); 1
2) ( 0 1 2 3 4 5 6 7 8 9 10 ); 10; I=1
  1 - automorphism 1
  10 - automorphisms 1,2,3,4,5,6,7,8,9,10

n = 12
1) ( 0  0  0  0  0  0  0  0  0  0  0  0 ); 1
2) ( 0  1  2  3  4  5  6  7  8  9 10 11 ); 4; I=1
3) ( 0  2  4  6  8 10  0  2  4  6  8 10 ); 2
4) ( 0  3  6  9  0  3  6  9  0  3  6  9 ); 2
5) ( 0  4  8  0  4  8  0  4  8  0  4  8 ); 2
6) ( 0  6  0  6  0  6  0  6  0  6  0  6 ); 1
  1 - automorphism 1
  2 - automorphisms 1,5
  4 - automorphisms 1,5,7,11

n = 13
1) ( 0  0  0  0  0  0  0  0  0  0  0  0  0 ); 1
2) ( 0  1  2  3  4  5  6  7  8  9 10 11 12 ); 12; I=1
  1 - automorphism 1
  12 - automorphisms 1,2,3,4,5,6,7,8,9,10,11,12

n = 14
1) ( 0  0  0  0  0  0  0  0  0  0  0  0  0  0 ); 1
2) ( 0  1  2  3  4  5  6  7  8  9 10 11 12 13 ); 6; I=1
3) ( 0  2  4  6  8 10 12  0  2  4  6  8 10 12 ); 6
4) ( 0  7  0  7  0  7  0  7  0  7  0  7  0  7 ); 1
  1 - automorphism 1
  6 - automorphisms 1,3,5,9,11,13

n = 15
1) ( 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0 ); 1
2) ( 0  1  2  3  4  5  6  7  8  9 10 11 12 13 14 ); 8; I=1
3) ( 0  3  6  9 12  0  3  6  9 12  0  3  6  9 12 ); 4
4) ( 0  5 10  0  5  0  0  5 10  0  5 10  0  5 10 ); 2
  1 - automorphism 1
  2 - automorphisms 1,2
  4 - automorphisms 1,2,4,8
  8 - automorphisms 1,2,4,7,8,11,13,14
```

```
n = 16
1) ( 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0 ); 1
2) ( 0  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 ); 8; I=1
3) ( 0  2  4  6  8 10 12 14  0  2  4  6  8 10 12 14 ); 4
4) ( 0  4  8 12  0  4  8 12  0  4  8 12  0  4  8 12 ); 2
5) ( 0  8  0  8  0  8  0  8  0  8  0  8  0  8  0  8 ); 1
  1 - automorphism 1
  2 - automorphisms 1,3
  4 - automorphisms 1,3,5,7
  8 - automorphisms 1,3,5,7,9,11,13,15

n = 17
1) ( 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0 ); 1
2) ( 0  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 ); 16; I=1
  1 - automorphism 1
  16 - automorphisms 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16

n = 18
1) ( 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0 ); 1
2) ( 0  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 ); 6; I=1
3) ( 0  2  4  6  8 10 12 14 16  0  2  4  6  8 10 12 14 16 ); 6
4) ( 0  3  6  9 12 15  0  3  6  9 12 15  0  3  6  9 12 15 ); 2
5) ( 0  9  0  9  0  9  0  9  0  9  0  9  0  9  0  9  0  9 ); 1
6) ( 0  6 12  0  6 12  0  6 12  0  6 12  0  6 12  0  6 12 ); 2
  1 - automorphism 1
  2 - automorphisms 1,5
  6 - automorphisms 1,5,7,11,13,17

n = 19
1) (0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0); 1
2) (0  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18); 18; I=1
  1 - automorphism 1
  19 - automorphisms 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19

n = 20
1) ( 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0 ); 1
2) ( 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 ); 8; I=1
3) ( 0  2  4  6  8 10 12 14 16 18  0  2  4  6  8 10 12 14 16 18 ); 4
4) ( 0  4  8 12 16  0  4  8 12 16  0  4  8 12 16  0  4  8 12 16 ); 4
5) ( 0  5 10 15  0  5 10 15  0  5 10 15  0  5 10 15  0  5 10 15 ); 2
6) ( 0 10  0 10  0 10  0 10  0 10  0 10  0 10  0 10  0 10  0 10 ); 1
  1 - automorphism 1
  2 - automorphisms 1,3
  4 - automorphisms 1,3,7,9
  8 - automorphisms 1,3,7,9,11,13,17,19
```

```
n = 21
1) ( 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ); 1
2) ( 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 ); 12; I=1
3) ( 0 3 6 9 12 15 18 0 3 6 9 12 15 18 0 3 6 9 12 15 18 ); 6
4) ( 0 7 14 0 7 14 0 7 14 0 7 14 0 7 14 0 7 14 0 7 14 ); 2
  1 - automorphism 1
  2 - automorphisms 1,2
  6 - automorphisms 1,2,4,5,10,13
  12 - automorphisms 1,2,4,5,8,10,11,13,16,17,19,20

n = 22
1) ( 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ); 1
2) ( 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 ); 10; I=1
3) ( 0 2 4 6 8 10 12 14 16 18 20 0 2 4 6 8 10 12 14 16 18 20 ); 10
4) ( 0 11 0 11 0 11 0 11 0 11 0 11 0 11 0 11 0 11 0 11 0 11 ); 1
  1 - automorphism 1
  10 - automorphisms 1,3,5,7,9,13,15,17,19,21

n = 23
1) (0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0); 1
2) (0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22); 22; I=1
  1 - automorphism 1
  22 - automorphisms 1,2,3,4,5,6,7,8,9,10,11,12,
    13,14,15,16,17,18,19,20,21,22
```

The notation `I=`i after the non-isomorphic distributive near-ring means that the near-ring has identity equal to i.

A technical mistake in [3, p.409] is found.

The near-ring "`33) (4,4,4,4,4,1)`" on $\mathbb{Z}_6$ is not distributively generated.

5. Conclusion

All classes of isomorphic near-rings on $\mathbb{Z}_n$, $n \leq 23$ are computed. These near-rings are checked for the right distributive law and investigated for some other properties.

In the future we will examine other properties of the computed near-rings on cyclic groups of order ≤ 23, such as regularity, strong regularity, N-regularity, π-regularity, quasiregularity and so on.

Acknowledgements

This research has been partially supported by project No IS-M-4/2008 of the Department for Scientific Research, Plovdiv University “Paisii Hilendarski”.

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Received 01 February 2010

НАМИРАНЕ НА КЛАСОВЕ ОТ ИЗОМОФНИ ПОЧТИ-ПРЪСТЕНИ НАД ЦИКЛИЧНИ ГРУПИ ОТ РЕД $\leq$ 23

Ангел Голев, Асен Рахнев

Резюме. Намерени са всички неизоморфни почти-пръстени над $\mathbb{Z}_n$, $n \leq 23$ и са изследвани за дистрибутивност и някои други свойства.