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NON-LYAPUNOV STABILITY ANALYSIS OF LARGE-SCALE SYSTEMS ON TIME-VARYING SETS IN TERMS OF TWO MULTI-VALUED MEASURES

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Abstract. Non-Lyapunov (h_0, h)-stability analysis of time-dependent non-linear large-scale systems of arbitrary order and structure is presented. The paper develops algebraic conditions for various types of practical and finite-time stability of the systems in terms of two multi-valued measures. The stability properties are studied on products of time-varying sets. The conditions guarantee a stability property of the overall system to be implied by the corresponding stability property of all subsystems.

Application of the aggregation-decomposition approach to the stability analysis reduces the dimension of the overall system aggregate matrix to the number of subsystems.

Key words: stability in terms of two multi-valued measures, large-scale systems

Mathematics Subject Classification 2000: 34D20

1. Introduction

In general, Lyapunov stability analysis fails to guarantee the required closeness. For the reasons mentioned above LaSalle and Lefschetz [7] emphasized the importance of non-Lyapunov stability investigations and developed such investigations. Meanwhile, their results are restricted by the requirement for a positive definiteness property of a system aggregate function.

Weiss and Infante [14] generalized essentially the Lyapunov method by allowing a system aggregate function and its Eulerian derivative to be signindefinite. Weiss and Infante [14, 15] prove sufficient conditions for different types of practical and finite-time stability. Gunderson [5] presents a significant analysis of the concept explained above.

In papers [6] and [8] set stability and uniform set stability of systems of ordinary differential equations (ODE) involve known specific bounds on solutions of systems of ODE under consideration. "Comparison theorems" are presented given sufficient conditions for these forms of stability.

Another substantial development of the non-Lyapunov stability theory is presented by Michel [9, 10] by relating a stability property to given time-varying sets. In the same papers practical stability and finite-time stability of composite systems are studied. The class of the systems is restricted to feedback systems with totally stable subsystems interconnected in cascade. Michel and Porter [11] further broaden the approach to the analysis of practical stability of discontinuous systems.

In order to realize good performances of automatic control systems such as those of trajectory control of space vehicles, frequency and voltage control in power systems, air-conditioning systems, temperature and pressure control in power plants and chemical processes one should assure their practical stability with the prescribed settling time. The analysis of practical stability with the settling time has been initiated by Grujic [1, 2]. In papers [3, 4] analysis of practical stability with the settling time is generalized to that on time-varying sets.

In this paper analysis of (h_0, h)-practical stability with the settling time is generalized to that on time-varying sets. Sufficient conditions for this stability are given.

2. System description

A composite system S to be considered is governed by the vector differential equation

$$\dot{x} = f(t, x, z), \tag{2.1}$$

where $x(t) \in R^n$ is the state of the system at time $t \in R^+, R^+ = [0, +\infty)$, $z : R^+ \times R^n \to R^m$ is a disturbance vector and $f : R^+ \times R^n \times R^m \to R^n$ is assumed to satisfy the adequate smoothness requirements so that solutions of (2.1) denoted by $x(t) = x(t; t_0, x_0),\ x(t_0; t_0, x_0) = x_0$ exist and are unique and continuous with respect to $t \in J$, and initial data. With J is denoted the time interval $[t_0, t_0 + \tau)$ where $t_0 \in R^+$ and $\tau \in R^+$, so that (h_0, h)-practical stability and (h_0, h)-finite-time stability will be studied simultaneously. In general, it is not required that $f(t,0,0) \equiv 0$, which means that the origin of the state space is not required to be an equilibrium state.

System S, eqn. (2.1), is composed of s interconnected subsystems S_i given by

$$\dot{x_i} = g_i(t,x_i) + r_i(t,x,z) \quad \text{for each} \quad i = 1,2,\dots,s, \tag{2.2}$$

where $g_i : R^+ \times R^{n_i} \to R^{n_i}$. Function $g_i(t,x_i)$ is supposed to satisfy the usual smoothness conditions so that solutions $x_i(t)=x_i(t;t_0,x_{i0})$, $x_i(t_0;t_0,x_{i0})=x_{i0}$ of the free subsystem S_i represented by

$$\dot{x_i} = g_i(t,x_i) \quad \text{for each} \quad i = 1,2,\dots,s \tag{2.3}$$

possess the same properties as solutions of the overall system S. Vector function $r_i : R^+ \times R^n \times R^m \to R^{n_i}$ of (2.2) is referred to as an interaction. The state vector $x_i \in R^{n_i}$ of S_i is the i-th component of the state vector x of the overall system, $x = (x_1^\tau, x_2^\tau, \dots, x_s^\tau)^\tau$.

Let us list the following classes of functions:

$K = \{w \in C[R^n, R^+] : w(s)$ is monotone increasing on $s, w(0) = 0$ and $w(s) \to \infty$ as $s \to \infty$,

$$\Gamma = \{h \in C[J \times R^n, R^n] : \inf_{x \in R^n} h(t,x) = 0 \quad \text{for each} \quad t \in J\}.$$

Function $h \in \Gamma$ is composed of $h^i(t,x_i)$, $h(t,x) = (h^{1T}(t,x_1), h^{2T}(t,x_2), \dots, h^{ST}(t,x_S))^T$.

Time-varying sets used in the sequel are assumed to possess the following properties. Set $\zeta_{(.)}(t)$ is a supposed to be open, connected and bounded set and $\zeta_{(.)}(t) \subset R^{n_{(.)}}$ for each $t \in J$. The closure and boundary of $\zeta_{(.)}(t)$ are denoted by $\overline{\zeta_{(.)}}(t)$ and $\partial\zeta_{(.)}(t)$, respectively. We accept that $\zeta_{(.)}(t)$ is continuous on J (see [3]),

$$\zeta_{(.)}(t) \in C(J). \tag{2.4}$$

Let $\zeta_A(t)$ be a given set of all allowable system states at arbitrary moment $t \in J \backslash J_s$ and let $\zeta_F(t)$ be a given set of all allowable states at $t \in J_s$, where $J_s = (t_0 + \tau_s, t_0 + \tau)$ and $J \backslash J_s = [t_0, t_0 + \tau_s]$. With $\tau_s, \tau_s \in [0,\tau]$, is denoted the system settling time, which is either prespecified or should be calculated. $\zeta_{Ii}(t_0)$ will be a set of all allowable $h_0^i(t_0,x_{i0})$ of $\boldsymbol{S_i}$ and S_i at time $t_0 \in R^+$. Set $\zeta_{Ai}(t), \zeta_{Ai}(t) \supseteq \zeta_{Ii}(t)$, will be a set of all allowable $h^i(t,x_i(t))$ at time $t \in J$ and $\zeta_D(t)$ will be a set of all allowable disturbances z. It is accepted that

$h^i(t, x_i(t)) \in \zeta_{Ai}(t)$ for each $i = 1, 2, \dots, s$ implies $z(t, x) \in \zeta_D(t)$ for each $t \in J$.

Further, with $\zeta_L(t), \zeta_L(t) \subseteq \zeta_F(t)$ for each $t \in J_s$ is denoted an open set such that

$$V_{ML}(t) \le V^{\partial}_{mL}(t) = V_{\overline{mc}}(t) \text{ for each } t \in J_s, \tag{2.5}$$

where $\zeta_c(t) = \zeta_A(t) \backslash \zeta_L(t)$.

A tentative aggregate function $\vartheta_i : R^+ \times R^{n_i} \to R^+$, which is associated with subsystems S_i and $\boldsymbol{S_i}$ is assumed differentiable, $\vartheta_i(t, x_i) \in C^{(1,1)}(D)$, $D = \{(t, x) : h(t, x) \in [\overline{\zeta_A}(t) \backslash (\zeta_L(t) \bigcap \zeta_I(t_0))]\}$. The Eulerian derivative of $\vartheta_i(t, x_i)$ along motions of S_i (2.3) is given by

$$\dot{\vartheta}_i = \frac{\partial \vartheta_i}{\partial t} + (\operatorname{grad} \vartheta_i)^T g_i \text{ for each } i = 1, 2, \dots, s \tag{2.6}$$

and along motions of S_i (2.2) by

$$\dot{\vartheta}_i = \dot{\vartheta}_i + (\operatorname{grad} \vartheta_i)^T r_i \text{ for each } i = 1, 2, \dots, s \tag{2.7}$$

A vector function $\boldsymbol{V} : R^+ \times R^n \to R^s$ associated with the overall system is composed of ϑ_i,

$$\boldsymbol{V} = (\vartheta_1, \vartheta_2, \dots, \vartheta_s)^T, \boldsymbol{V}^{(.)}_{(..)} = (\vartheta^{(.)}_{i(..)}). \tag{2.8}$$

The following usual notations will be used throughout the paper.

$$\begin{cases} \vartheta_{iM(.)}(t) = \sup[\vartheta_i(t, x_i) : h^i(t, x_i) \in \zeta_{(.)i}(t)], \\ \vartheta_{\overline{iM(.)}}(t) = \sup[\vartheta_i(t, x_i) : h^i(t, x_i) \in \overline{\zeta_{(.)i}}(t)], \\ \vartheta_{im(.)}(t) = \inf[\vartheta_i(t, x_i) : h^i(t, x_i) \in \zeta_{(.)i}(t)], \\ \vartheta^{\partial}_{im(.)}(t) = \inf[\vartheta_i(t, x_i) : h^i(t, x_i) \in \partial_{\zeta_{(.)i}}(t)], \\ \vartheta_{\overline{im(.)}}(t) = \inf[\vartheta_i(t, x_i) : h^i(t, x_i) \in \overline{\zeta_{(.)i}}(t)], \\ \vartheta(t, x) = \sum\limits_{i=1}^{s} \vartheta_i(t, x_i). \end{cases}$$

With $\|x\|$ is denoted the Euclidean norm of vector x and $\varnothing$ is the vacuous set.

3. Preliminaries

In this paper we shall refer to

Definition 3.1. System (2.1) is (h_0, h)-practically stable with the settling time τ_s with respect to $\{t_0,\ J,\ \zeta_I(t_0),\ \zeta_A(t),\ \zeta_F(t),\ \zeta_D(t)\}$ if and only if $h_0(t_0, x_0) \in \zeta_I(t_0)$ and $z(t,x) \in \zeta_D(t)$ for each $(t,x) \in J \times R^n$ for which $h(t,x) \in \zeta_A(t)$ imply $h(t, x(t; t_0, x_0)) \in \zeta_A(t_0)$ for each $t \in J$ and $h(t, x(t; t_0, x_0)) \in \zeta_F(t_0)$ for each $t \in J_s$.

Referring to Russinov [12, 13] we can easily verify

Theorem 3.1. *System (2.1) is (h_0, h)-practically stable with the settling time τ_s with respect to $\{t_0, J, \zeta_I(t_0), \zeta_A(t), \zeta_F(t), \zeta_D(t)\}$ if $h(t,x) \in \overline{\zeta_A}(t)$ implies $z(t,x) \in \overline{\zeta_D}(t)$ for each $t \in J$ and if there exist a function $\vartheta \in R^+ \times R^n \to R^+$, $\vartheta(t,x) \in C^{(1,1)}(D)$, a function $\varphi : R^+ \to R^+$, which is integrable over J, a function $\theta \in K$ and function $h_0, h \in \Gamma$, so that*

$$(3.1)\qquad h(t, x(t)) \le \theta(h_0(t, x(t)))e, \quad \text{whenever} \quad h_0(t, x(t)) \in \zeta_I^*(t),$$

where $e \in R^n, e = (1, 1, \ldots, 1)$ and $\zeta_I^*(t) \subset R^n$ for each $t \in J$,

$$(3.2)\qquad \dot{\vartheta}(t, x, z) < \varphi(t) \quad \text{for each} \quad (t, x) \in D \quad \text{and for each} \quad z \in \zeta_D(t),$$

$$(3.3)\qquad \int_{t_0}^{t} \varphi(\sigma)\mathrm{d}\sigma \le {\vartheta_{mA}}^{\partial}(t) - \vartheta_{\overline{MI}}(t_0) \quad \text{for each} \quad t \in J \backslash J_s$$

and

$$(3.4)\qquad \int_{t_0}^{t} \varphi(\sigma)\mathrm{d}\sigma < {\vartheta_{mL}}^{\partial}(t) - \vartheta_{\overline{MI}}(t_0) \quad \text{for each} \quad t \in J_s.$$

4. Main Results

In the subsequent development vector function $r : R^+ \times R^n \times R^m \to R^n$ is supposed to belong either to class C_1 or C_2, where

$$(4.1)\quad C_1 = \{r : (\operatorname{grad} \vartheta_i)^T r_i \le \\ \le \sum_{j=1}^{s} \xi_{ij} \varphi_j(t),\ \forall (t,x) \in D,\ \forall z \in \overline{\zeta_D}(t),\ \forall i = 1, 2, \ldots, s\},$$

$$(4.2)\quad C_2 = \{r : (\operatorname{grad}\vartheta_i)^T r_i \leq \leq \sum_{j=1}^{s} \xi_{ij\partial}\psi_i(t)\psi_j(t),\ \forall(t,x)\in D,\ \forall z \in \overline{\zeta_D}(t),\ \forall i = 1,2,\dots,s\},$$

All ξ_{ij} are real numbers. Function $r(t,x,z)$ is composed of $r_i(t,x,z)$

$$(4.3)\qquad r_i = (r_1^T, r_2^T, \dots, R_s^T)^T.$$

Functions ψ_i are supposed to satisfy the following conditions for some real numbers μ_i and ν_i

$$(4.4)\quad \mu_i \leq \varphi_i(t)\psi_i^{-2}(t) \leq \nu_i, \quad \text{for each} \quad t \in J \quad \text{and for each} \quad i = 1,2,\dots,s.$$

In the sequel every set $\zeta_{(.)}(t)$ without subscript $i, i = 1,2,\dots,s$, is accepted to represent the Cartesian product of all $\zeta_{(.)i}(t)$ and to be associated with the overall system S (2.1), as used in (4.1) and (4.2),

$$(4.5)\qquad \zeta_{(.)}(t) = \prod_{i=1}^{s} \zeta_{(.)i}(t).$$

Set $\zeta_{(.)i}(t)$ is related to the subsystem S_i for each $i = 1,2,\dots,s$. Further, we introduce functions $\theta_i : R^+ \times R^+ \to R^+$ and $\Theta : R^+ \times R^+ \to R^s$,

$$(4.6)\qquad \theta_i(t_0,t) = \int_{t_0}^{t} \varphi_i(\sigma)d\sigma,$$

$$(4.7)\qquad \Theta = (\theta_1, \theta_2, \dots, \theta_s)^T.$$

All functions $\theta_i(t_0,t)$ are assumed to be determined by verifying the conditions of Theorem 3.1, for the corresponding subsystems S_i. Besides, we shall use a real constant aggregate $s \times s$ matrix $A = (a_{ij})$, where

$$(4.8)\qquad a_{ij} = \delta_{ij} + \xi_{ij} \quad \text{for each} \quad i,j = 1,2,\dots,s$$

and δ_{ij} is the Kronecher symbol.

Theorem 4.1. *Let $h_0, h \in \Gamma$ and let each subsystem S_i (2.3) of the composite system S be (h_0^i, h^i)-practically stable with the settling time τ_s with respect to $\{t_0,\ J,\ \zeta_{Ii}(t),\ \zeta_{Ai}(t),\ \zeta_{Fi}(t),\ \varnothing\}$, which is proved by using Theorem 3.1.*

Then the overall system S (2.1) is (h_0, h)-practically stable with the same settling time τ_s with respect to $\{t_0,\ J,\ \zeta_I(t),\ \zeta_A(t),\ \zeta_F(t),\ \zeta_D(t)\}$, $\forall r \in C_1$ if

$$A\Theta(t_0, t) \le V_{mA}{}^{\partial}(t) - V_{\overline{MI}}(t_0),\ \forall t \in J\backslash J_s \tag{4.9}$$

and

$$A\Theta(t_0, t) \le V_{mL}{}^{\partial}(t) - V_{\overline{MI}}(t_0),\ \forall t_0 \in J_s, \tag{4.10}$$

where matrix $A = (a_{ij})$ is determined by (4.8).

Proof. The theorem will be proved by contradiction. Let the conditions of the theorem be satisfied and yet system S (2.1) not be (h_0, h)-practically stable with the settling time τ_s. The assumption allows the existence of $t_1 \in J\backslash J_s$ and $i \in [1, s]$ such that

$$\begin{aligned} &h^i(t, x_i(t_1; t_0, x_{i0})) \in \partial\zeta_{Ai}(t_1), \\ &\qquad h_0^i(t_0, x_{i0}) \in \zeta_{Ii}(t_0),\ z \in \zeta_D(t),\ \forall t \in [t_0, t_1]. \end{aligned} \tag{4.11}$$

Let time t_1 be the first moment satisfying (4.11). For each interconnected subsystem S_i we may write

$$\vartheta_i[t, x_i(t; t_0, x_{i0})] = \vartheta_i(t_0, x_{i0}) + \int_{t_0}^{t} \dot{\vartheta}_i[\sigma; x_i(\sigma; t_0, x_{i0})]d\sigma\,. \tag{4.12}$$

The stability property of each free subsystem S_i is established by utilizing Theorem 3.1. Therefore, (3.1), (3.2) and (3.3) hold for all free subsystems. Using (3.2), (3.3), (4.1), (4.6)–(4.8) and (4.12) we get $\vartheta_i[t_1, x_i(t_1; t_0, x_{i0})] < \vartheta_{\overline{iMIi}}(t_0) + \sum\limits_{j=1}^{s} a_{ij}\theta_j(t_0, t)$.

This result, together with (4.9), implies $\vartheta_i[t_1, x_i(t_1; t_0, x_{i0})] < \vartheta_{imAi}{}^{\partial}(t_1)$, which can be valid only if $h^i(t_1, x_i(t_1; t_0, x_{i0})) \notin \partial\zeta_{Ai}(t_1)$. But, this contradicts the original assumption (4.11). Therefore, $h^i(t, x_i(t; t_0, x_{i0})) \in \zeta_{Ai}(t)$, $\forall t \in J\backslash J_s$, for each $(t_0, x_0) \in J \times R^n$ for which $h^i(t_0, x_{i0}) \in \zeta_{Ii}(t_0)$, $\forall z \in \zeta_D(t),\ \forall i = 1, 2, \ldots, s$.

If we again suppose that system S is not (h_0, h)-practically stable with the settling time τ_s despite the conditions of the theorem, then there could exist $t_2 \in J_s$ and $i \in [1, s]$ so that

$$h^i(t_2, x_i(t_2; t_0, x_{i0})) \in \partial\zeta_{Ai}(t_2),\quad h_0^i(t_0, x_{i0}) \in \zeta_{Ii}(t_0). \tag{4.13}$$

Let $t_2 \in J_s$ be the first moment for which (4.13) is valid. Using (2.5), (3.1)–(3.4) applied to all subsystems S_i, (4.1), (4.6)–(4.10) and (4.12) we get $\vartheta_i[t_2, x_i(t_2; t_0, x_{i0})] \leq {\vartheta_{imLi}}^{\partial}(t_2) \leq \vartheta_{\overline{imCi}}(t_2), t_2 \in J_s$.

Therefore, $h^i(t_2, x_i(t_2; t_0, x_{i0})) \notin \partial\zeta_{Ai}(t_2), t_2 \in J_s$ which disproves (4.13) and demonstrates that $h(t, x(t; t_0, x_0)) \notin \zeta_A(t)$ for each $t \in J$. This result and $\vartheta_i[t_1, x_i(t; t_0, x_{i0})] \in {\vartheta_{imLi}}^{\partial}(t) = \vartheta_{\overline{imCi}}(t)$ imply $h^i(t, x_i(t; t_0, x_{i0})) \in \zeta_L(t) \subseteq \zeta_F(t)$, $\forall h_0^i(t_0, x_{i0}) \in \zeta_{Ii}(t_0), \forall z \in \zeta_D(t), \forall t \in J_s$, which completes the proof. □

Corollary 4.1. *Let $h_0, h \in \Gamma$ and let each subsystem S_i (2.3) of the composite system S (2.1) be (h_0^i, h^i)-practically stable with respect to $\{t_0,\ J,\ \zeta_{Ii}(t_0),\ \zeta_{Ai}(t)\}$, which is proved by using Theorem 3.1 for the case $\zeta_{Fi}(t) \equiv \zeta_{Ai}(t)$, $\zeta_D(t) \equiv \varnothing$, $\tau_s = \tau$, $J_s = \varnothing$. Then the overall system S (2.1) is totally stable with respect to $\{t_0,\ J,\ \zeta_I(t_0),\ \zeta_A(t),\ \zeta_D(t)\}$, $\forall r \in C_1$ if*

$$A\Theta(t_0, t) \leq {V_{mA}}^{\partial}(t) - V_{\overline{MI}}(t_0), \forall t \in J \tag{4.14}$$

provided that matrix $A = (a_{ij})$ is determined by (4.8).

In some cases the composite system S is not disturbed by vector z, i.e. $\zeta_D(t) \equiv \varnothing$. Then it can be significant to analyze the (h_0, h)-practical stability of the system S with respect to $\{t_0,\ J,\ \zeta_I(t_0),\ \zeta_A(t)\}$.

Corollary 4.2. *Let $h_0, h \in \Gamma$ and let each subsystem S_i (2.3) of the composite system S be (h_0^i, h^i)-practically stable with respect to $\{t_0, J, \zeta_{Ii}(t_0), \zeta_{Ai}(t)\}$, which is proved by using Theorem 3.1 for the case $\zeta_{Fi}(t) \equiv \zeta_{Ai}(t)$, $\zeta_D(t) \equiv \varnothing$, $\tau_s = \tau$, $J_s = \varnothing$. Then the overall system S (2.1) is (h_0, h)-practically stable with respect to $\{t_0,\ J,\ \zeta_I(t_0),\ \zeta_A(t)\}$, $\forall r \in C_1$ if (4.14) hold, provided that matrix $A = (a_{ij})$ is determined by (4.8).*

Example 4.1. Composite system S of dimension $n = 6$ consists of three second order ($n_i = 2, \forall i = 1, 2, 3$) subsystems S_i described by

$$\dot{x_i} = A(t)x_i, \quad A(t) = \begin{pmatrix} -3.1(1+t)^{-1} & 2 \\ -2 & -3.1(1+t)^{-1} \end{pmatrix}.$$

System interactions r_i are given by

$$\begin{aligned} r_1(t, x) &= \frac{1}{2(1+t)^2} \begin{pmatrix} \mathrm{sat}(x_{21} + x_{31}) \\ \mathrm{sat}(2x_{22} + x_{32}) \end{pmatrix}, \\ r_2(t, x) &= \frac{1}{2(1+t)^2} \begin{pmatrix} \mathrm{sat}(0.1x_{11} + 0.4x_{32}) \\ \mathrm{sat}(0.7x_{12} + 0.7x_{31}) \end{pmatrix}, \\ r_3(t, x) &= \frac{1}{2(1+t)^2} \begin{pmatrix} \mathrm{sat}(x_{12} + x_{21}) \\ \mathrm{sat}(x_{11} + x_{22}) \end{pmatrix}, \end{aligned}$$

where $\operatorname{sat}\rho = \left\{\begin{array}{l} \operatorname{sign}\rho,\ |\rho| \geq 1 \\ \rho,\ |\rho| \leq 1 \end{array}\right.$.

Let $h_0^i(t, x_i) = h^i(t, x_i) = \|x_i\|,\ i = 1, 2, 3$. We want to test (h_0, h)-practical stability with the settling time $\tau_s = (e^{\frac{1}{2}} - 1)$ of composite system S given above with respect to $\{0, J, \zeta_I(0), \zeta_A(t), \zeta_F(t), \varnothing\}$, where $J = [0, e^4 - 1)$,

$$\zeta_{Ii}(0) = \{(t, x_i) : h_0^i(0, x_i) < e^{1/2}\},\ \zeta_{Ai}(t) = \{(t, x_i) : h^i(t, x_i) < (e/(1{+}t))\},$$

$$\zeta_{(.)}(t) = \prod_{i=1}^{3} \zeta_{(.)i}(t),\quad \zeta_{Fi}(t) = \{(t, x_i) : h^i(t, x_i) < (1/(1{+}t))\},$$

$$D_i = \{(t, x_i) : (1/(1+t)) \leq h^i(t, x_i) \leq (e/(1+t))\},\quad D = \prod_{i=1}^{3} D_i.$$

Sign-indefinite function $\vartheta_i(t, x_i) = 2\ln(1 + t).\|x_i\| \in C^{(1.1)}(D)$ is a candidate aggregate function of subsystems $S_i, \forall i = 1, 2, 3$. At first we determine $V_{\overline{Mi}}(0) = (1, 1, 1, 1)^\tau; V_{\overline{mA}}{}^{\partial}(t) = (2, 2, 2, 2)^\tau, \forall t \in J$.

Further, $\zeta_{Li}(t) \equiv \zeta_{Fi}(t)$ is accepted so that $V_{mL}{}^{\partial}(t) = 0,\ V_{\overline{mC}}(t) = V_{\overline{mL}}{}^{\partial}(t),\ \forall t \in J$.

The last equation shows that all $\vartheta_i(t, x_i)$ satisfy (2.5). Since

$$\dot{\vartheta}_i(t, x_i) = -(4.2/(1 + t)),\ \forall h^i(t, x_i) \in \overline{\zeta}_{Ai}(t),\ \forall t \in J$$

we can set $\varphi_i(t) = -4(1{+}t)^{-1},\ \forall i = 1, 2, 3$. Therefore $\theta_i(0, t) = -4\ln(1{+}t)$, $\Theta(0, t) = -4(1, 1, 1)^\tau \ln(1 + t),\ \Theta(0, t) < (1, 1, 1)^\tau = V_{mA}{}^{\partial}(t) - V_{\overline{MI}}(0)$, $\forall t \in J\backslash J_s,\ \Theta(0, t) < -(2, 2, 2)^\tau < -(1, 1, 1)^\tau = V_{mA}{}^{\partial}(t) - V_{\overline{MI}}(0),\ \forall t \in J_s$ which proves (Theorem 3.1) (h_0^i, h^i)-practical stability with the settling time $\tau_s = (e^{1/2}{-}1)$ of each subsystem S_i with respect to $\{0, J, \zeta_{Ii}(0), \zeta_{Ai}(t), \zeta_{Fi}(t), \varnothing\}$. Further

$$\operatorname{grad}\vartheta_i = 2x_i\|x_i\|^{-2},\ \forall i = 1, 2, 3$$

and

$$(\operatorname{grad}\vartheta_1)^T r_1 \leq \frac{1}{4}\varphi_1(t) - \frac{1}{8}[\varphi_2(t) + \varphi_3(t)],\quad \forall (t, x) \in D,$$

$$(\operatorname{grad}\vartheta_2)^T r_2 \leq -\frac{1}{8}\varphi_1(t) - \frac{1}{4}\varphi_2(t) - \frac{1}{8}\varphi_3(t),\ \forall (t, x) \in D,$$

$$(\operatorname{grad}\vartheta_3)^T r_3 \leq -\frac{1}{8}[\varphi_1(t) + \varphi_2(t)] - \frac{1}{4}\varphi_3(t),\ \forall (t, x) \in D,$$

so that $\xi_{11} = -\frac{1}{4},\ \xi_{12} = \xi_{13} - \frac{1}{8},\ \xi_{21} = -\frac{1}{8},\ \xi_{22} = -\frac{1}{4},\ \xi_{23} = -\frac{1}{8}$, $\xi_{31} = \xi_{32} = -\frac{1}{8},\ \ \xi_{33} = -\frac{1}{4}$.

Matrix A (4.8) is now obtained as

$$A = \frac{1}{8}\begin{pmatrix} 6 & -1 & -1 \\ -1 & 6 & -1 \\ -1 & -1 & 6 \end{pmatrix}$$

so that

$$A\Theta(0,t) = -\frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \ln(1+t)^4 < \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = V_{mA}^{\partial}(t) - V_{\overline{MI}}(0),\ \forall t \in (J \backslash J_s),$$

$$A\Theta(0,t) < -\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = V_{mL}{}^{\partial}(t) - V_{\overline{MI}}(0),\ \forall t \in J_s.$$

These results demonstrate that all conditions of Theorem 4.1 are satisfied. The composite system is (h_0, h)-practically stable with the settling time $\tau_s = (e^{1/2} - 1)$ with respect to $\{0,\ J,\ \zeta_I(0),\ \zeta_A(t),\ \zeta_F(t),\ \varnothing\}$.

In the sequel all aggregate functions $\varphi_i(t)$ are assumed to be sign-semidefinite,

$$\varphi_i(t)\left\{ \begin{array}{l} \geq 0,\ \forall i = 1, 2, \ldots, k \\ \leq 0,\ \forall i = k+1, k+2, \ldots, s,\ \forall t \in J \end{array} \right\}. \tag{4.15}$$

Functions $\psi_i(t)$ are supposed to obey

$$|\psi_i(t)| = [\varepsilon_i \varphi_i(t)]^{1/2},\ \forall i = 1, 2, \ldots, s, \tag{4.16}$$

where $\varepsilon_i = \left\{ \begin{array}{r} 1,\ \forall i = 1, 2, \ldots, k \\ -1,\ \forall i = k+1, k+2, \ldots, s. \end{array} \right.$

In what follows we shall use a constant vector

$$\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_s)^T. \tag{4.17}$$

Besides, we introduce a symmetric $s \times s$ matrix $B = (b_{ij})$ with constant elements b_{ij} determined by

$$b_{ij} = \varepsilon_i \delta_{ij} + \frac{1}{2}(\xi_{ij} + \xi_{ji}),\ \forall i, j = 1, 2, \ldots, s, \tag{4.18}$$

where ξ_{ij} are given by (4.2). With $\Lambda(B)$ will be denoted the maximal eigenvalue of the matrix B. Using the previous notations we can state a criterion for (h_0, h)-practical stability with the settling time of composite systems on product spaces.

Theorem 4.2. *Let $h_0, h \in \Gamma$ and let each subsystem S_i (2.3) of composite system S (2.1) be (h_0^i, h^i)-practically stable with the settling time τ_s with respect to $\{t_0,\ J,\ \zeta_{Ii}(t_0),\ \zeta_{Ai}(t),\ \zeta_{Fi}(t),\ \varnothing\}$, which is proved by using Theorem 3.1, and let all functions $\varphi_i(t)$ of the same theorem be sign-semidefinite (4.15). Then the overall system S (2.1) is (h_0, h)-practically stable with the same settling time τ_s with respect to $\{t_0, J, \zeta_I(t_0), \zeta_A(t), \zeta_F(t), \zeta_D(t)\}, \forall r \in C_2$ (4.2), (4.16) if the following hold:*

$$\Lambda(B)\varepsilon^T\Theta(t_0,t) \leq {\vartheta_{mA}}^{\partial}(t) - \vartheta_{\overline{MI}}(t_0),\ \forall t \in J \backslash J_s,$$

$$\Lambda(B)\varepsilon^T\Theta(t_0,t) \leq {\vartheta_{mL}}^{\partial}(t) - \vartheta_{\overline{MI}}(t_0),\ \forall t \in J_s$$

provided the elements b_{ij} of the matrix B are determined by (4.2) and (4.18), where $\vartheta = \sum\limits_{i=1}^{s} \vartheta_i$.

If we accept $\varphi(t) = \Lambda(B) \sum\limits_{j=1}^{s} \varepsilon_j \varphi_j(t)$, then we easily prove Theorem 4.2 as a consequence of Theorem 3.1. Theorem 4.2 can also be used to test other (h_0, h)-practical stability and (h_0, h)-finite-time stability properties of composite systems.

Example 4.2. Subsystems $S_i (n_i = 2)$ are described by

$$\dot{x_i} = (1+t)^3 A_i x_i, \quad Ai = \begin{pmatrix} -12.1 & 4 \\ 0 & -16.1 \end{pmatrix}.$$

System interactions are defined by

$$\begin{aligned} r_1(t,x) &= \begin{pmatrix} -2(1{+}t)^4 \operatorname{sat} 0.1x_{11} + 0.5\alpha(x_{11}{+}x_{22}) \\ -2(1{+}t)^4 \operatorname{sat} 0.1x_{12} + 0.5\alpha(x_{12}{+}x_{21}) \end{pmatrix}, \\ r_2(t,x) &= 0.5 \begin{pmatrix} \beta(x_{12}) \\ \beta(x_{11}) \end{pmatrix}, \end{aligned}$$

where non-linearities $\alpha(\zeta)$ and $\beta(\zeta)$ are depicted in Figure 1.

Let $h_0^i(t, x_i) = h^i(t, x_i) = \|x_i\|,\ i = 1, 2$. We want to test the (h_0, h)-practical stability with the settling time $\tau_s = 2$ of the composite system S defined above with respect to $t_0 = 0,\ \tau = +\infty$, and the products (4.5) of sets

$$\zeta_{Ii}(0) = \{(t, x_i) : h_0^i(0, x_i) < 2\},$$

$$\zeta_{Ai}(t) = \{(t, x_i) : h^i(t, x_i) < \frac{10}{1+t}\},$$

$$\zeta_{Fi}(t) = \{(t, x_i) : h^i(t, x_i) < \frac{1}{1+t^2}\},$$
$$D_i = \{(t, x_i) : \frac{1}{1+t^2} \le h^i(t, x_i) \le \frac{10}{1+t}\}.$$

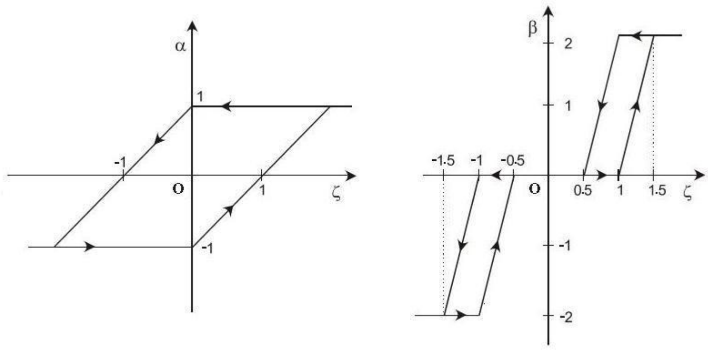

Figure 1. Non-linearities of Example 4.2

Function $\vartheta_i(t, x_i) = \|x_i\|$, $i = 1, 2$, is chosen as a candidate aggregate function of subsystem S_i. Then we select $\zeta_{Li}(t) \equiv \zeta_{Fi}(t)$, and find

$$\dot{\vartheta}_i(t, x_i) < -2(1+t) = \varphi_i(t), \quad \theta_i(0, t) = -(2t + t^2),$$
$$\forall i = 1, 2, \ \forall h^i(t, x_i) \in \overline{\zeta}_{Ai}(t), \ \forall t \in J,$$

$$V_{MI}(0) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \quad V_{mA}{}^{\partial}(t) = \frac{10}{1+t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad V_{mL}{}^{\partial}(t) = \frac{1}{1+t^2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

So that and $V_{mL}{}^{\partial}(t) \equiv V_{\overline{mC}}(t)$ and

$$\Theta(0, t) = -(2t + t^2)\begin{pmatrix} 1 \\ 1 \end{pmatrix} < \frac{8 - 2t}{1+t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = V_{mA}{}^{\partial}(t) - V_{\overline{MI}}(0), \forall t \in J \backslash J_s,$$

$$\Theta(0, t) = -(2t + t^2)\begin{pmatrix} 1 \\ 1 \end{pmatrix} < -\frac{1 + 2t^2}{1+t^2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = V_{mL}{}^{\partial}(t) - V_{\overline{MI}}(0), \forall t \in J_s.$$

These results prove (Theorem 3.1) that subsystem S_i is (h_0^i, h^i)-practically stable with the settling time $\tau_s = 2$ with respect to $\{0, J, \zeta_{Ii}(0), \zeta_{Ai}(t), \zeta_{Fi}(t), \varnothing\}$, $\forall i = 1, 2$.

Further, we find and $\operatorname{grad}\vartheta_i = x_i\|x_i\|^{-1}$ and

$$(\operatorname{grad}\vartheta_1)^T r_1 \le -0.1\psi_1^2(t) + 0.5\psi_1(t)\psi_2(t),\ \forall (t,x)\in D,$$

$$(\operatorname{grad}\vartheta_2)^T r_2 \le \psi_1(t)\psi_2(t),\ \forall (t,x)\in D,$$

$$\psi_i(t) = [2(1+t)]^{1/2},\quad \forall i=1,2;\ \xi_{11}=-0.1,\ \xi_{12}=0.5,\ \xi_{21}=1,\ \xi_{22}=0.$$

Matrix B (4.18) is now obtained as

$$B = \begin{pmatrix} -1.1 & 0.75 \\ 0.75 & -1 \end{pmatrix},\quad \Lambda(B) = -0.59.$$

Vector ε is given by $\varepsilon = (-1,-1)^T$ so that

$$\Lambda(B)\varepsilon^T\Theta(0,t) = -1.18(2t+t^2)$$

and

$$\Lambda(B)\varepsilon^T\Theta(0,t) < \begin{cases} \frac{2(8-2t)}{1+t} = {\vartheta_{mA}}^{\partial}(t) - \vartheta_{\overline{MI}}(0),\ \forall t\in J\backslash J_s \\ -\frac{2(1+2t^2)}{1+t^2} = {\vartheta_{mL}}^{\partial}(t) - \vartheta_{\overline{MI}}(0),\ \forall t\in J_s, \end{cases}$$

where $\vartheta = \sum\limits_{i=1}^{2}\vartheta_i$.

All conditions of Theorem 4.2 are satisfied and we may conclude that the composite system S is (h_0,h)-practically stable with the settling time $\tau_s = 2$ with respect to $\{0,\ J,\ \zeta_I(0),\ \zeta_A(t),\ \zeta_F(t),\ \varnothing\}$.

In some cases conditions (4.16) can be restrictive. In order to relax conditions imposed of the system we shall present another criterion for its (h_0,h)-practical stability with the settling time. Let

$$c = (c_1,c_2,\dots,c_s)^T,\quad c_j = \sum_{i=1}^{s} a_{ij},\quad \forall j=1,2,\dots,s, \tag{4.19}$$

where are determined by (4.8).

Theorem 4.3. *Let $h_0, h\in\Gamma$ and let each subsystem S_i (2.3) of the large-scale system S (2.1) be (h_0^i,h^i)-practically stable with the settling time τ_s with respect to $\{t_0,\ J,\ \zeta_{Ii}(t_0),\ \zeta_{Ai}(t),\ \zeta_{Fi}(t),\ \varnothing\}$, which is proved by using Theorem 3.1. Then the overall system S (2.1) is (h_0,h)-practically stable with* *the same settling time* τ_s *with respect to* $\{t_0,\, J,\, \zeta_I(t_0),\, \zeta_A(t),\, \zeta_F(t),\, \zeta_D(t)\}$, $\forall r \in C_1$, *if*

$$c^T \Theta(t_0, t) \leq {\vartheta_{mA}}^{\partial}(t) - \vartheta_{\overline{MI}}(t_0), \quad \forall t \in J \backslash J_s,$$

$$c^T \Theta(t_0, t) \leq {\vartheta_{mL}}^{\partial}(t) - \vartheta_{\overline{MI}}(t_0), \quad \forall t \in J_s,$$

where vector c *is determined by (4.19) and* $\vartheta = \sum_{i=1}^{s} \vartheta_i$.

Theorem 4.3 is easily verified by referring to Theorem 3.1.

Example 4.3. System S is composed of three subsystems S_i described by

$$\dot{x_i} = A_i(t) x_i, \quad A_i(t) = \begin{pmatrix} -18(1+t) & 2\sin t \\ -2\sin t & -18(1+t) \end{pmatrix}, \ \forall i = 1, 2, 3$$

and interactions

$$\begin{aligned} r_1(t,x) &= \frac{1+t}{4} \begin{pmatrix} x_{11} & x_{21} \\ x_{12} & x_{31} \end{pmatrix}, \\ r_2(t,x) &= \frac{0.1}{1+t} \begin{pmatrix} x_{31} \\ x_{11} \end{pmatrix}, \\ r_3(t,x) &= \frac{1}{40} \begin{pmatrix} x_{11} & x_{21} \\ x_{12} & x_{22} \end{pmatrix}. \end{aligned}$$

Let $h_0^i(t, x_i) = h^i(t, x_i) = \|x_i\|$. We want to test the (h_0, h)-practical stability with the settling time $\tau_s = 9$ of the composite systems S if $t_0 = 0$, $\tau = +\infty$,

$$\begin{aligned} \zeta_{Ii}(0) &= \{(t, x_i) : h^i(0, x_i) < 10\}, \\ \zeta_{Ai}(t) &= \{(t, x_i) : h^i(t, x_i) < \frac{10}{(1+t)^2}\}, \\ \zeta_{Fi}(t) &= \{(t, x_i) : h^i(t, x_i) < \frac{1}{(1+t)^2}\} \end{aligned}$$

and

$$D_i = \{(t, x_i) : \frac{1}{(1+t)^2} \leq h_i(t, x_i) \leq \frac{10}{(1+t)^2}\}.$$

At first we test the same stability property of all subsystems. Function $\vartheta_i(t,x_i) = \ln(1+t)^2\|x_i\|$ is proposed as a candidate aggregate function for subsystems S_i. We can choose $\zeta_{Li}(t) \equiv \zeta_{Fi}(t)$. It is found

$$\vartheta_{\overline{iMIi}}(0) = \ln 10, \quad {\vartheta_{imAi}}^{\partial} = \ln 10, \quad {\vartheta_{imLi}}^{\partial}(t) \equiv \vartheta_{\overline{imCi}}(t) = 0$$

and

$$\vartheta_i(t,x_i) < \varphi_i(t), \quad \varphi_i(t) = -16(1+t)^{-1}$$

for each (t,x_i) for which $h^i(t,x_i) \in \zeta_{Ai}(t)$, $\forall t \in J$, which implies $\theta_i(0,t) = -16\ln(1+t)$. With this result we can test the (h_0^i, h^i)-practical stability with the settling time $\tau_s = 9$ of all subsystems. It is obtained

$$\theta_i(0,t) = -16\ln(1+t) \le \begin{cases} 0 = {\vartheta_{imAi}}^{\partial}(t) - \vartheta_{\overline{iMIi}}(0), \forall t \in J \backslash J_s \\ -\ln 10 = {\vartheta_{imLi}}^{\partial}(t) - \vartheta_{\overline{iMIi}}(0), \forall t \in J_s, \forall i{=}1,2,3. \end{cases}$$

Applying Theorem 3.1, all conditions of which are satisfied, we conclude that each subsystem $S_i,\ i=1,2,3$, is (h_0^i, h^i)-practically stable with the settling time $\tau_s = 9$ with respect to $\{0,\ J,\ \zeta_{Ii}(0),\ \zeta_{Ai}(t),\ \zeta_{Fi}(t),\ \varnothing\}$. To test the (h_0, h)-stability property of the overall system we find

$$\operatorname{grad}\vartheta_i = x_i\|x_i\|^{-2}$$

$$(\operatorname{grad}\vartheta_1)^T r_1 \le -\frac{5}{32}\varphi_2(t) - \frac{5}{32}\varphi_3(t), \forall (t,x) \in D,$$

$$(\operatorname{grad}\vartheta_2)^T r_2 \le -\frac{5}{32}\varphi_1(t) - \frac{5}{32}\varphi_3(t), \forall (t,x) \in D,$$

$$(\operatorname{grad}\vartheta_3)^T r_3 \le -\frac{5}{32}\varphi_1(t) - \frac{5}{32}\varphi_2(t), \forall (t,x) \in D.$$

Elements a_{ij} (4.8) of the matrix A are easily calculated as follows:

$$a_{11} = 1,\ a_{12} = \frac{5}{32},\ a_{13} = -\frac{5}{32},\ a_{21} = -\frac{5}{32},$$
$$a_{22} = 1,\ a_{23} = -\frac{5}{32},\ a_{31} = -\frac{5}{32},\ a_{32} = -\frac{5}{32},\ a_{33} = 1.$$

Vector c (4.19) is given by

$$c = \frac{11}{16}(1,1,1)^T.$$

Since

$$c^T\Theta(0,t) = -33\ln(1+t) \le 0, \ \forall t \in J,$$

$${\vartheta_{mA}}^{\partial}(t) = 3\ln 10, \ \forall t \in J, \ {\vartheta_{\overline{MI}}}^{\partial}(0) = 3\ln 10 \ \text{ and } \ {\vartheta_{mL}}^{\partial}(t) = 0, \ \forall t \in J$$

we can easily verify all conditions of Theorem 4.3,

$$c^T\Theta(0,t) = -33\ln(1+t) \le \begin{cases} 0 = {\vartheta_{mA}}^{\partial}(t) - \vartheta_{\overline{MI}}(0), \ \forall t \in J\backslash J_s \\ -3\ln 10 = {\vartheta_{mL}}^{\partial}(t) - \vartheta_{\overline{MI}}(0), \ \forall t \in J_s, \end{cases}$$

where $\vartheta = \sum_{i=1}^{3} \vartheta_i$.

These results prove that the system S is (h_0, h)-practically stable with the settling time $\tau_s = 9$ with respect to $\{0,\ J,\ \zeta_I(0),\ \zeta_A(t),\ \zeta_F(t),\ \varnothing\}$.

5. Conclusions

Algebraic criteria have been developed for various types of (h_0, h)-practical stability and (h_0, h)-finite-time stability of large-scale systems on product spaces. To apply the conditions it is necessary to prove (h_0^i, h^i)-practical stability or (h_0^i, h^i)-finite-time stability of all subsystems and to use information about interactions. Such an aggregate-decomposition approach has resulted in the reduction of the dimension of the system aggregate matrix to the number of subsystems. Furthermore, no restriction is imposed on either system dimensionality or its structure.

The stability analysis in terms of two multi-valued measures has been carried out on products of time-varying sets, which provides information about the trajectory bounds and the settling time of the overall system.

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АНАЛИЗ НА НЕЛЯПУНОВА УСТОЙЧИВОСТ НА СИСТЕМИ С ГОЛЯМА РАЗМЕРНОСТ СПРЯМО МНОЖЕСТВА, ЗАВИСЕЩИ ОТ ВРЕМЕТО, ПО ОТНОШЕНИЕ НА ДВЕ МНОГОМЕРНИ МЕРКИ

Иван К. Русинов

Резюме. Представя се анализ на неляпунова (h_0, h)-устойчивост на нелинейни системи с голяма размерност от произволен ред и структура, които зависят от времето. В статията са получени алгебрични условия за различни видове практическа устойчивост и устойчивост на краен интервал на системи по отношение на две многомерни мерки. Свойствата на устойчивостта се изследват спрямо многомерни множества, зависещи от времето. Условията осигуряват дадено свойство за устойчивост на цялата система да следва от съответното свойство за устойчивост на всички подсистеми.

Приложението на подхода на агрегиране и декомпозиране към анализа на устойчивостта намалява размерността на агрегираната матрица на цялата система до броя на подсистемите.