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STRIPED NETS IN A THREE-DIMENSIONAL SPACE OF WEYL

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Abstract. Striped nets in a two-dimensional Riemannian space are introduced and studied by Stauber [3] and Komisaruk [1]. Properties of some special striped nets are found in [2]. B. Tsareva in [4] and [5] defines and studies striped nets in a two-dimensional space of Weyl.

Striped nets in a three-dimensional space of Weyl are defined and studied in this paper.

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***Key words*:** prolonged covariant differentiate, derivative equations, striped net, chebyshevian net.

1. Preliminaries

Let in a three-dimensional space of Weyl $W_3(g_{is}, T_k)$ with a fundamental tensor g_{is} and an additional vector T_k, be given three independent fields of directions $\underset{\alpha}{v}^i$. There is a net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}) \in W_3$, defined by the independent fields of directions $\underset{\alpha}{v}^i$ for $\alpha = 1, 2, 3$. The reciprocal covectors $\overset{\alpha}{v}_i$ of $\underset{\alpha}{v}^i$ are defined by the equations:

$$\underset{\alpha}{v}^i \overset{\alpha}{v}_k = \delta_k^i \Leftrightarrow \underset{\alpha}{v}^i \overset{\beta}{v}_i = \delta_\alpha^\beta \tag{1}$$

We standardize the fields of directions $\underset{\alpha}{v}^i$ by equation [6]

$$g_{is} \underset{\alpha}{v}^i \underset{\alpha}{v}^s = 1. \tag{2}$$

If $\underset{\alpha\beta}{\omega}$ is the angle between the fields of directions $\underset{\alpha}{v}^i$ and $\underset{\beta}{v}^i$, then following [6] we have

$$g_{is}\underset{\alpha}{v}^i\underset{\alpha}{v}^s = \cos \underset{\alpha\beta}{\omega}. \tag{3}$$

In [6] there is introduced the prolonged covariant differentiate of the satellites of the fundamental tensor g_{is} with weight $\{k\}$. From [6] we have

$$\dot{\nabla}_k g_{is} = 0,\ \dot{\nabla}_k g^{is} = 0. \tag{4}$$

There $\dot{\nabla}$ is the symbol of the prolonged covariant derivative, and g^{is} is the reciprocal tensor of g_{is}. In [6], the following derivative equations are worked out:

$$\dot{\nabla}_k \underset{\alpha}{v}^i = \underset{\alpha}{\overset{\sigma}{T}}_k \underset{\sigma}{v}^i,\ \dot{\nabla}_k \overset{\alpha}{v}_i = -\underset{\sigma}{\overset{\alpha}{T}}_k \overset{\sigma}{v}_i,\ \sigma = 1, 2, 3. \tag{5}$$

From (1) and (3) we have $g_{ik}\underset{\alpha}{v}^i = \overset{\beta}{v}_k \cos \underset{\alpha\beta}{\omega}$. From (4) and the last equation after the prolonged covariant derivative and contracting by $\underset{\beta}{v}^v$ we obtain

$$\underset{\alpha}{\overset{\sigma}{T}}_j \cos \underset{\sigma v}{\omega} + \underset{v}{\overset{\sigma}{T}}_j \cos \underset{\sigma\alpha}{\omega} = \left(\cos \underset{\alpha v}{\omega}\right)_j. \tag{6}$$

2. Striped nets in W_3

2.1. Striped nets of first kind

Definition 1. The net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}) \in W_3$ will be called a striped net of first kind if

$$\underset{\alpha\beta}{\omega}_j = \lambda \overset{\alpha}{v}_j + \mu \overset{\alpha}{v}_j, \tag{7}$$

where $\alpha \neq \beta$ and $\alpha, \beta = 1, 2, 3$.

From the definition it follows that the gradient of the net angle $\underset{\alpha\beta}{\omega}_j$ belongs to the platform, defined by the covectors $\overset{\alpha}{v}_j$ and $\overset{\alpha}{v}_j$.

Proposition 1. *The net* $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}) \in W_3$ *is striped of first kind if and only if:*

$$(8) \qquad \left(\overset{\sigma}{\underset{\alpha}{T}}{}_j \cos \underset{\sigma\beta}{\omega} + \overset{\sigma}{\underset{\beta}{T}}{}_j \cos \underset{\sigma\alpha}{\omega}\right)\underset{\gamma}{v}{}^j = 0, \ (\alpha, \beta, \gamma) = (1,2,3), (2,3,1), (3,1,2).$$

Proof. Let $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}) \in W_3$ be a striped net of first kind. From (6) and (7) we obtain

$$(9) \qquad \overset{\sigma}{\underset{\alpha}{T}}{}_j \cos \underset{\sigma\beta}{\omega} + \overset{\sigma}{\underset{\beta}{T}}{}_j \cos \underset{\sigma\alpha}{\omega} = \underset{(\alpha)}{\lambda} \overset{(\alpha)}{v}{}_j + \underset{\alpha}{\mu} \overset{\beta}{v}{}_j, \ (\alpha, \beta) = (1,2), (2,3), (3,1).$$

From here, after contracting by $\underset{\gamma}{v}{}^k$ we obtain the equations (8).
(The branched indexes are not to be summed.)

Conversely, let equations (8) be satisfied for a net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}) \in W_3$, then equations (9) follow easily which shows that the net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v})$ is a striped net one of first kind. □

2.2. Striped Nets of second kind

Definition 2. The net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}) \in W_3$ will be called a striped one of second kind if:

$$(10) \qquad \underset{\alpha\beta}{\omega}{}_j = \underset{\alpha}{\lambda}(\overset{\alpha}{v}{}_j + \overset{\beta}{v}{}_j) + \underset{\alpha}{\mu} \overset{\gamma}{v}{}_j, \ (\alpha, \beta, \gamma) = (1,2,3), (2,3,1), (3,2,1).$$

Proposition 2. *The net* $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}) \in W_3$ *be striped of second kind if and only if:*

$$(11) \qquad \left(\overset{\sigma}{\underset{1}{T}}{}_j \cos \underset{\sigma 2}{\omega} + \overset{\sigma}{\underset{2}{T}}{}_j \cos \underset{\sigma 1}{\omega}\right)\left(\underset{1}{v}{}^j - \underset{2}{v}{}^j\right) = 0,$$
$$(\alpha, \beta, \gamma) = (1,2,3), (2,3,1), (3,2,1).$$

Proof. Let the net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}) \in W_3$ be a striped of second kind. From (6) and (10) we obtain:

$$(12) \qquad \overset{\sigma}{\underset{\alpha}{T}}{}_j \cos \underset{\sigma\beta}{\omega} + \overset{\sigma}{\underset{\beta}{T}}{}_j \cos \underset{\sigma\alpha}{\omega} = \underset{(\alpha)}{\lambda}\left(\overset{(\alpha)}{v}{}_j + \overset{\beta}{v}{}_j\right) + \underset{\alpha}{\mu} \overset{\gamma}{v}{}_j,$$
$$(\alpha, \beta, \gamma) = (1,2,3), (2,3,1), (3,2,1).$$

From here, after contracting by $\underset{1}{v}^j - \underset{2}{v}^j$, $\underset{2}{v}^j - \underset{3}{v}^j$ and $\underset{3}{v}^j - \underset{1}{v}^j$ respectively we obtain (11).

Conversely, let equations (11) be valid for the net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}) \in W_3$. From (6) and (11) we obtain (10), i.e. the net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v})$ is a striped one of second kind. □

3.1. Striped Nets

Definition 3. The net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}) \in W_3$ will be called striped, if it is a striped net of first and second kind.

From Proposition 1 and Proposition 2 it follows:

Proposition 3. *The net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}) \in W_3$ is striped if and only if:*

$$\begin{gathered}\left(\overset{\sigma}{\underset{\alpha}{T}}_j \cos \underset{\sigma\beta}{\omega} + \overset{\sigma}{\underset{\beta}{T}}_j \cos \underset{\sigma\alpha}{\omega}\right)\left(\underset{\alpha}{v}^j - \underset{\beta}{v}^j + \underset{\gamma}{v}^j\right) = 0, \\ (\alpha, \beta, \gamma) = (1,2,3), (2,3,1), (3,2,1). \end{gathered} \tag{13}$$

Corollary 1. *The striped net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}) \in W_3$ is a geodesic net if and only if*

$$\begin{gathered}\overset{\sigma}{\underset{\alpha}{T}}_k \underset{\sigma}{v}^k = 0, \quad \alpha \neq \sigma, \\ \overset{\sigma}{\underset{\alpha}{T}}_k \cos \underset{\sigma\beta}{\omega}(\underset{\gamma}{v}^k - \underset{\phi}{v}^k) + \overset{\sigma}{\underset{\beta}{T}}_k \cos \underset{\sigma\varepsilon}{\omega}(\underset{1}{v}^k - \underset{\gamma}{v}^k) = 0, \\ (\alpha, \beta, \gamma) = (1,2,3), (2,3,1), (3,2,1). \end{gathered} \tag{14}$$

Proof. Following [6], we have

$$\overset{\sigma}{\underset{\alpha}{T}}_k \underset{\sigma}{v}^k = 0, \ \alpha \neq \sigma. \tag{15}$$

From (15) and (13) we obtain (14). The converse is also true. If (14) is valid for the striped net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v})$, the is a geodesic one. □

Corollary 2. *The striped net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}) \in W_3$ is chebyshevian of first kind if and only if:*

$$\overset{\sigma}{\underset{\alpha}{T}}_k \underset{\sigma}{v}^k = 0, \ \overset{\sigma}{\underset{\alpha}{T}}_k \underset{\alpha}{v}^k \cos \underset{\sigma\beta}{\omega} - \overset{\sigma}{\underset{\beta}{T}}_k \underset{\beta}{v}^k \cos \underset{\sigma\alpha}{\omega} = 0, \ \alpha \neq \beta; \ \alpha, \beta = 1,2,3. \tag{16}$$

Proof. Chebyshevian nets of second kind are characterized by equations [6]:

$$\overset{\sigma}{\underset{\alpha}{T}}{}_k \underset{\beta}{v}{}^k = 0,\ \alpha \neq \sigma;\ \alpha, \beta = 1, 2, 3 \tag{17}$$

From (17) and (13) we obtain (16). The converse is also true. If (16) is valid for the striped net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v})$, the net is a chebishevian one of second kind. □

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ИВИЧНИ МРЕЖИ В ТРИМЕРНОТО ПРОСТРАНСТВО НА ВАЙЛ

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Резюме. В работата са дефинирани и изследвани ивични мрежи в тримерно вайлово пространство W_3. Получени са характеристики на тези мрежи, които се изразяват с зависимости между полетата от направления на мрежата, коефициентите на деривационните уравнения на мрежата и ъгъла между полетата от направления. Намерени са необходими и достатъчни условия дадена ивична мрежа да е геодезична или чебишева от втори род.